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Publication date:
2011

[Link to publication in Tilburg University Research Portal](#)

Citation for published version (APA):

Cherchye, L. J. H., Demuynck, T., & de Rock, B. (2011). *Noncooperative Household Consumption with Caring*. (CentER Discussion Paper; Vol. 2011-019). Econometrics.

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No. 2011-019

**NONCOOPERATIVE HOUSEHOLD CONSUMPTION WITH
CARING**

By Laurens Cherchye, Thomas Demuynck and Bram de Rock

10 February 2011

ISSN 0924-7815

Noncooperative household consumption with caring^{*}

Laurens Cherchye[†], Thomas Demuynck[‡] and Bram De Rock[§]

February 10, 2011

Abstract

We present a household consumption model that accounts for caring household members, while allowing for noncooperative behavior in decisions on public goods. The intrahousehold consumption outcome critically depends on the degree of caring between the household members. By varying the degree of intrahousehold caring, the model encompasses a whole continuum of household consumption models that are situated between the fully cooperative model and the noncooperative model without caring. This feature is used to define a measure for the degree of cooperation within the household. We also establish a dual characterization of our noncooperative model with caring preferences: we show that the model is dually equivalent to a noncooperative model with non-caring preferences that is characterized by intrahousehold transfers. Finally, following a revealed preference approach, we derive testable implications of the model for empirical data. We demonstrate the practical usefulness of the model through an illustrative application.

JEL Classification: D11, D12, D13, C14.

Keywords: household consumption, caring preferences, intrahousehold cooperation, Nash equilibrium, revealed preferences.

^{*}We thank seminar participants in Oxford for useful discussion. The usual disclaimer applies.

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1 Introduction

Household members care for each other. But, at the same time, they may act non-cooperatively when deciding on the publicly consumed goods within the household. How can we account for this in modeling household consumption behavior? We present a consumption model that can allow for various degrees of caring in the household, while considering possibly noncooperative behavior. More specifically, we assume that household members have caring preferences in the Beckerian sense (also referred to as altruistic preferences by Becker (1981)). We then model noncooperative behavior by assuming that households choose Nash equilibrium intrahousehold allocations.

Our model fits within the so-called ‘non-unitary’ approach to analyzing household consumption behavior, which has become increasingly popular in the recent literature. Indeed, there is a growing consensus that multi-member consumption behavior should no longer be modeled as resulting from the maximization of some common household welfare function. This ‘unitary’ approach to modeling household behavior is methodologically unappealing and leads to testable implications (e.g. income pooling and Slutsky symmetry) that are frequently rejected in empirical studies.¹

Non-unitary household consumption models open the ‘black box’ of household behavior by taking into account that each household member has her/his own preferences. Consumption decisions are then regarded as the outcome of specific intrahousehold decision processes. In our non-unitary model, the outcome of the household decision process critically depends on the degree of caring between the household members. By varying the degree of intrahousehold caring, the model encompasses a whole continuum of household consumption models that are situated between the fully cooperative model (with Pareto efficient intrahousehold allocations) and the noncooperative model without caring (with Nash equilibrium allocations under non-caring preferences). As such, our model provides a generalized perspective on modeling household consumption with public goods. As we will discuss in Section 2, the cooperative model and the noncooperative model without caring have been well-documented in the literature. Each model has its own strengths and weaknesses. A main objective of the current study is to develop a consumption model that combines the attractive properties of the cooperative and noncooperative benchmark models, while avoiding the associated weaknesses.

Our consumption model has a number of additional features that are particularly

¹Alderman, Chiappori, Haddad, Hoddinot and Kanbur (1995), Vermeulen (2002) and Donni (2008) provide more elaborate discussions of this topic. For empirical rejections of the unitary model, see for example Lundberg (1988), Thomas (1990), Bourguignon, Browning, Chiappori and Lechene (1993), Lundberg, Pollak and Wales (1993), Fortin and Lacroix (1997), Phipps and Burton (1998), Browning and Chiappori (1998), Chiappori, Fortin and Lacroix (2002), Duflo (2003), Vermeulen (2005) and Cherchye and Vermeulen (2008).

attractive from a theoretical and/or practical perspective. First of all, as we will argue in Section 3, it allows us to define a measure of intrahousehold caring that can also be interpreted as quantifying the degree of within-household cooperation. Specifically, we show that it is possible to quantify and estimate the degree of caring within the household; and this gives us an operational measure for the magnitude of intrahousehold cooperation. We see at least two reasons why it is important to know this degree of intrahousehold cooperation. First, from a welfarist perspective, it gives us an idea of the welfare improvement that is possible within a certain household. If it is possible to link the level of cooperation to household characteristics, we may use this knowledge for welfare enhancement measures that correct the efficiency loss originating from household behavior that is not fully cooperative. Second, the extent of within-household cooperation is also important for the structure of optimal taxation and policies that target to alter the intrahousehold income distribution.² In this respect, different (cooperative-noncooperative) consumption models may lead to other intrahousehold allocations.

Another interesting feature of our model pertains to its dual representation, which will be established in Section 4. Specifically, we will show that the noncooperative model with caring preferences is dually equivalent to a noncooperative model with non-caring preferences that is characterized by intrahousehold transfers. In fact, the intrahousehold transfers in the dual model will be directly related to the above mentioned measure of intrahousehold cooperation. This duality result parallels the well-known duality between a Pareto optimal allocation and the Lindahl equilibrium, which is often used to provide a decentralized representation for the fully cooperative (Pareto efficient) model of household consumption. As such, we obtain a similar decentralized representation for our newly proposed model.

A final important aspect of our model relates to its empirical applicability. In Section 5 we will show that, although our newly proposed model generalizes the fully cooperative and noncooperative models, it does have useful testable implications for empirical data. To this end, we present a revealed preference characterization of the model in the tradition of Afriat (1967) and Varian (1982): we derive necessary and sufficient conditions for the empirical validity of our model that can be checked by solely using a finite set of observed household consumption bundles and corresponding prices.³ Essentially, this revealed preference characterization directly applies the theoretical implications of our consumption model to the observed household choices.

²See, for example, Blundell, Chiappori and Meghir (2005) for discussion of this targeting view on tax policies. Browning, Bourguignon, Chiappori and Lechene (1994), Browning, Chiappori and Lewbel (2006) and Lewbel and Pendakur (2008) focus on alternative welfare-related questions associated with the intrahousehold income distribution in the context of the cooperative consumption model.

³See also Samuelson (1938), Houthakker (1950) and Diewert (1973) for seminal contribution to the revealed preference approach to modeling household consumption behavior.

In our opinion, this makes it a natural starting point for investigating the empirical usefulness of this newly proposed model. In this respect, we also indicate that the revealed preference approach has been successfully applied for empirical analysis of non-unitary consumption models: Cherchye, De Rock and Vermeulen (2007, 2009, 2011) focus on the cooperative model, while Cherchye, Demuynck and De Rock (2011) consider the noncooperative model without caring. In addition, as we will discuss below, this revealed preference approach has some attractive advantages (as compared to the more standard ‘differential’ approach) for analyzing multi-member household consumption behavior. We will demonstrate the practical usefulness of the revealed preference conditions by means of an illustrative application to data taken from the Russia Longitudinal Monitoring Survey (RLMS).

The concluding Section 6 will summarize our main results. In addition, it will suggest a number of interesting avenues for follow-up research.

2 Non-unitary models of household consumption: overview

Within the non-unitary approach, alternative household consumption models differ from each other in their modeling of the intrahousehold decision process. In particular, we distinguish two main approaches in the existing literature. The first approach assumes that the household members behave cooperatively, which means that they reach a Pareto-optimal allocation, i.e. no household member can increase her/his utility without decreasing the utility of any other member.⁴ The second approach assumes noncooperative behavior and excludes intrahousehold caring, i.e. the household consumption allocation is a Nash equilibrium defined in terms of non-caring preferences.⁵ In a household consumption setting with both privately and publicly consumed goods, this implies a Nash equilibrium with household members voluntarily contributing to the public goods. It is well known that, in this case, the resulting level of public goods is generally below the cooperative (Pareto efficient) level.

Both the cooperative model and the noncooperative model have their own strengths and weaknesses. The defense of the noncooperative model without caring is almost entirely based on its theoretical appeal. In particular, any Nash equilibrium is stable

⁴See, for example, Apps and Rees (1988), Chiappori (1988, 1992), Browning and Chiappori (1998) and Cherchye, De Rock and Vermeulen (2007, 2011). Following Chiappori (1988, 1992), the consumption literature often refers to the cooperative model as the ‘collective’ model of household behavior.

⁵See, for example, Leuthold (1968), Bourguignon (1984), Ulph (1988), Kooreman and Kapteyn (1990), Browning (2000), Chen and Woolley (2001), Lechene and Preston (2005, 2008), Browning, Chiappori and Lechene (2010) and Cherchye, Demuynck and De Rock (2011).

in the sense that no household member can increase her/his utility by unilaterally changing her/his strategy. Moreover, using a backward induction argument, one can show that this stability property remains even if we allow for finitely repeated interaction.

Nevertheless, the noncooperative approach also has some deficiencies. First of all, it seems rather unrealistic –especially in a household setting– to assume that household members only care about their own wellbeing. This calls for including caring preferences. Second, the household is normally viewed as a prime example of an institution that it is very likely to overcome free-rider problems associated with public consumption –at least to some extent. Specifically, one may expect that repeated interaction and (nearly) perfect information increase the probability that household members develop welfare enhancing mechanisms to overrule such problems.

Let us then consider the cooperative model. The premise of efficient behavior can be defended in three ways (see, for example, Browning and Chiappori (1998)). First of all, under perfect information and with repeated interactions –two conditions that are likely to be satisfied within every household– Pareto optimal allocations can be stable as long as all members are sufficiently patient. Second, the Pareto outcome is seen as a most natural generalization of the assumption of utility maximization in the unitary model with several household members. Finally, Pareto efficiency is widely used as an assumption in cooperative bargaining models.⁶ In this sense, Pareto optimality is a minimal condition that should be satisfied if the intrahousehold bargaining process is based on such a cooperative solution concept.

Although we largely agree with these arguments, we also believe that there remains scope for relaxing the efficiency condition. First of all, it is well known that, unless the Pareto optimal allocation exactly coincides with a Nash equilibrium, the cooperative Pareto efficient outcome is not self enforcing. In other words, there will usually be some household member(s) who can increase utility by unilaterally deviating from the Pareto optimal allocation. Second, even if we are in a situation with infinitely repeated interaction, the folk theorem shows that almost every allocation situated between the noncooperative Nash outcome and the Pareto efficient outcome could be stable. In other words, (infinitely) repeated interaction does not necessarily lead to efficient behavior. Finally, the Pareto efficiency assumption has been questioned for the publicly consumed goods. Most notably, it has been argued that the informational requirement and the resulting cost of implementing cooperation may often be unrealistic.

Summarizing, while the fully cooperative model might represent an overly opti-

⁶See Manser and Brown (1980), McElroy and Horney (1981) and Lundberg and Pollak (1993) for applications of bargaining models in a household setting.

mistic outlook of the household decision process, we may also argue that the non-cooperative model without caring is too pessimistic. Indeed, it appears to us that most households are to be found somewhere between the cooperative and noncooperative benchmarks. As noted by Alderman, Chiappori, Haddad, Hoddinot and Kanbur (1995): ‘[The household] consists of individuals who – motivated at times by altruism, at times by self interest, and often by both — cajole, cooperate, threaten, help, argue, support, and, indeed, occasionally walk out on each other.’

In this paper, we present a new model of household behavior that encompasses situations between the extreme cases of full cooperation and noncooperation without caring. Formally, our model is equivalent to a noncooperative model where household members have Beckerian caring preferences: each household member optimizes a function that is increasing in the utilities of all household members.⁷ In this set-up, we will derive specific testable restrictions for empirical data. Interestingly, we will also demonstrate that it is possible to empirically recover a measure for the degree of intrahousehold cooperation which, as we will explain, actually captures caring within the household.

Thus, by introducing caring in the noncooperative framework, our model allows us to combine some attractive properties of the cooperative model and the noncooperative model. At the same time, it solves two main problems associated with the two benchmark models. First of all, as it is based on the concept of a non-cooperative Nash equilibrium, it is self enforcing and, hence, stable. Second, by introducing caring between the household members, we depart from the assumption that these members are inherently egoistic (i.e. non-caring). Caring preferences allow for friendship, altruism, love and trust between household members. We believe this assumption to be much more realistic when dealing with institutions like households, where these emotions do play an important role.

As a final remark, it is worth to note that d’Aspremont and Dos Santos Ferreira (2009) provide an alternative household consumption model that is situated between the fully cooperative and the noncooperative model. A most important difference with our model is that these authors model ‘semicooperative’ behavior by parameterizing the trade-off between an individual budget constraint and the household budget constraint (which evaluates the public goods at Lindahl prices). By contrast, the distinguishing feature of our approach is that it combines caring preferences with noncooperative intrahousehold interaction for modeling the household decision behavior. See also the concluding section for a further comparison between our model and the model of d’Aspremont and Dos Santos Ferreira (in terms of testable implications).

⁷In this respect, it is also worth referring to Browning and Lechene (2001), who adopt a similar approach to investigate the relationship between expenditures (on private and public goods) and the intrahousehold distribution of income.

3 A noncooperative model with caring preferences

We consider a household with two members, A and B .⁸ The household decides over the purchase of a bundle of N private goods, denoted by $\mathbf{q} \in \mathbb{R}_+^N$, and a bundle of K intrahousehold public goods, denoted by $\mathbf{Q} \in \mathbb{R}_+^K$. We remark that this assumes that each good is either private (in \mathbf{q}) or public (in \mathbf{Q}). Further, it excludes externalities associated with privately consumed quantities. Importantly, however, our setting can actually account for such externalities. Specifically, if an individual is the exclusive consumer of a particular private good, then we can account for externalities for this good by formally treating it as a public good. Throughout, we will treat the first private good as a numeraire and we will assume that the consumption of the numeraire and all public goods is strictly positive in all household equilibria.⁹

In what follows, we will first formalize our assumptions regarding the preferences and the strategies of the household members. Subsequently, we will formally define and characterize the household equilibrium in terms of our model.

Preferences: Our analysis starts from a set of decision situations T . In each situation t , the household faces a price vector $\mathbf{p}_t \in \mathbb{R}_{++}^N$ for the private goods, a price vector $\mathbf{P}_t \in \mathbb{R}_{++}^K$ for the public goods, and a household income $Y_t \in \mathbb{R}_{++}$. In addition, members A and B are endowed with situation-dependent concave and increasing (Beckerian) caring functions. We denote these functions by $W_t^A(U^A, U^B)$ and $W_t^B(U^B, U^A)$; in this construction, U^A and U^B stand for ‘egoistic’ utility functions which (only) depend on the members’ own consumption of private goods (\mathbf{q}^A and \mathbf{q}^B) and the total amount of public goods (\mathbf{Q}), i.e. $U^A = U^A(\mathbf{q}^A, \mathbf{Q})$ and $U^B = U^B(\mathbf{q}^B, \mathbf{Q})$. Of course, the vectors representing the individual consumption of the private goods should add up to the total household consumption of these goods, i.e. $\mathbf{q}^A + \mathbf{q}^B = \mathbf{q}$. In contrast to the caring functions W_t^A and W_t^B , we assume that the utility functions U^A and U^B are stable (invariant) across all decision situations t in T . Indeed, if these functions were also situation-dependent, then our model would have no testable implications. Further, we will assume that utility functions U^A and U^B are continuous, concave, non-satiated and non-decreasing in their arguments.

An important feature of our model is that the caring functions W_t^A and W_t^B are situation-dependent. This is a natural assumption in a non-unitary framework. Specifically, it reflects the idea that the degree of caring or altruism between household members might depend on several (situation-dependent) exogenous variables.¹⁰

⁸This focus on two-member households is mainly to keep the exposition simple. However, our following analysis can readily be extended to households with more than two members.

⁹We can relax this assumption by using suitable Lagrange multipliers, but this would only increase notational complexity without adding new insights. In fact, our own empirical application in Section 5 will consider data sets with some components of the public goods equal to zero.

¹⁰Compare with the discussion in Browning, Chiappori and Lechene (2006). These authors

These variables are analogous to the so-called extra-environmental parameters in the terminology of McElroy and Horney (1981) or distribution factors in the terminology of Browning, Bourguignon, Chiappori and Lechene (1994). They come in two kinds. On the one hand, exogenous variables may influence the decision process within the household. Examples of such variables are the state of the marriage market, the state of the labor market, the specific divorce laws and the social attitudes to the roles of men and women within the household. On the other hand, exogenous variables may impact on the emotional state of the household members. Examples of such variables are the amounts of love, friendship, compassion and trust within the household. Both kinds of variables may have a strong influence on the shape of the caring functions. Taking the caring functions to be situation-dependent allows the model to adapt to a change in each of these (typically unobserved) variables.

In what follows, we will make one additional assumption to facilitate our technical analysis. Specifically, we use a single crossing (SC) property:

Assumption SC: *For all decision situations t , $\mathbf{q}^A, \mathbf{q}^B \in \mathbb{R}^N$ and $\mathbf{Q} \in \mathbb{R}_+^K$, for $\bar{U}^A = U^A(\mathbf{q}^A, \mathbf{Q})$ and $\bar{U}^B = U^B(\mathbf{q}^B, \mathbf{Q})$, we have that either*

$$\left. \frac{\partial W_t^A}{\partial U^B} \right|_{(\bar{U}^A, \bar{U}^B)} = 0,$$

or

$$-\left. \frac{\frac{\partial W_t^A}{\partial U^A}}{\frac{\partial W_t^A}{\partial U^B}} \right|_{(\bar{U}^A, \bar{U}^B)} \leq -\left. \frac{\frac{\partial W_t^B}{\partial U^A}}{\frac{\partial W_t^B}{\partial U^B}} \right|_{(\bar{U}^B, \bar{U}^A)}.$$

The left hand side of the last inequality provides the amount of utility U^A that A is willing to subsume to compensate a one unit increase in U^B . In other words, it gives the slope of the indifference curve of the function W_t^A in \mathbb{R}^2 space through the point (\bar{U}^A, \bar{U}^B) , i.e. the marginal rate of substitution between U^A and U^B . Assumption SC states that, for every combination of utilities \bar{U}^A and \bar{U}^B , the slope of the indifference curve for W_t^A through this point is steeper than the slope of the indifference curve of W_t^B through this point. Intuitively, this single crossing condition implies that, when compared to member B , member A gives at least the same weight to her own utility U^A as to the utility of the other member U^B . Symmetrically, B gives relatively more weight to U^B than to U^A in comparison to A . We believe this to be an intuitively plausible assumption. Observe that Assumption SC is entirely ordinal. In other words, it is insensitive to any monotonic transformation of W_t^A, W_t^B, U^A or U^B .

Strategies: In order to combine noncooperation and caring in one and the same formal model, we make the following assumption regarding the household members' consider (situation-dependent) aggregation of preferences in a cooperative framework.

strategies. At every decision situation t , each household member decides on three bundles: member A chooses the private bundles $\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B} \in \mathbb{R}_+^N$ and the public bundle $\mathbf{Q}_t^A \in \mathbb{R}_+^K$; and, similarly, member B chooses $\mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A} \in \mathbb{R}_+^N$ and $\mathbf{Q}_t^B \in \mathbb{R}_+^K$. We interpret as follows. The bundle $\mathbf{q}_t^{A,A}$ is the bundle of private goods that member A buys for herself, $\mathbf{q}_t^{A,B}$ is the bundle of private goods that A buys for the other member B , and \mathbf{Q}_t^A is the contribution to the bundle of public goods purchased by A . The meaning of $\mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}$ and \mathbf{Q}_t^B is directly analogous. Of course, we must have $\mathbf{q}_t^{A,A} + \mathbf{q}_t^{B,A} = \mathbf{q}_t^A$, $\mathbf{q}_t^{B,B} + \mathbf{q}_t^{A,B} = \mathbf{q}_t^B$ and $\mathbf{Q}_t^A + \mathbf{Q}_t^B = \mathbf{Q}_t$.

It is standard in the literature on noncooperative household behavior to explicitly distinguish between A and B 's contribution to the household's public consumption (e.g. Lechene and Preston (2005, 2008), and d'Aspremont and Dos Santos Ferreira (2009) make similar distinctions). However, the fact that we allow A and B to buy private goods for each other may seem a bit unconventional. In most models (of noncooperative behavior) it is assumed that members only buy private goods for themselves, i.e. A chooses \mathbf{q}_t^A and B chooses \mathbf{q}_t^B . Our distinction between $\mathbf{q}_t^{M,M}$ and $\mathbf{q}_t^{M,L}$ (for $M, L \in \{A, B\}$, $M \neq L$) directly relates to the specificity of our model, i.e. it accounts for caring preferences in a noncooperative setting.

Let us explain this last point in some more detail. In a noncooperative model without caring preferences, it seems intuitive that individual members will not buy private goods for the other. By contrast, in the case of intrahousehold caring, one household member may well benefit from increasing the private consumption of the other member. Our distinction between $\mathbf{q}_t^{M,M}$ and $\mathbf{q}_t^{M,L}$ exactly takes this into account.¹¹ In fact, in many real life situations one household member effectively buys private consumption goods for the other member. Examples are abundant: the wife goes shopping and buys food for everyone and clothes for her husband; the husband fills the car with gasoline while the wife takes the car to go to the gym; etc.

Equilibrium: We will first introduce our new concept of household equilibrium in general terms. Subsequently, we will show that the concept encompasses the fully cooperative equilibrium and the noncooperative equilibrium without caring as limiting cases. This demonstrates the generality of our model. Furthermore, it will enable us to interpret our measure of intrahousehold caring as quantifying the degree of within-household cooperation, i.e. the measure allows us to distinguish between different consumption models characterized by different degrees of cooperation.

We assume that in equilibrium both members maximize their caring functions given the decisions of the other members, i.e. we assume a noncooperative Nash

¹¹Browning, Chiappori and Lechene (2010) suggest a similar idea in the context of a noncooperative model with one private good and one public good, where one individual has caring preferences while the other individual is egoistic. In fact, a similar mechanism also underlies Becker's rotten kid theorem.

equilibrium. More formally, at decision situation t , member A solves the following optimization problem (**OP-A**):

$$\begin{aligned}
(\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{Q}_t^A) &= \arg \max_{(\mathbf{q}^{A,A}, \mathbf{q}^{A,B}, \mathbf{Q}^A)} W_t^A(U^A(\mathbf{q}^A, \mathbf{Q}), U^B(\mathbf{q}^B, \mathbf{Q})) \\
\text{s.t. } &\mathbf{p}'_t(\mathbf{q}^A + \mathbf{q}^B) + \mathbf{P}'_t \mathbf{Q} \leq Y_t \\
&\mathbf{q}_t^{A,A} + \mathbf{q}_t^{B,A} = \mathbf{q}^A \\
&\mathbf{q}_t^{A,B} + \mathbf{q}_t^{B,B} = \mathbf{q}^B \\
&\mathbf{Q}^A + \mathbf{Q}_t^B = \mathbf{Q}
\end{aligned}$$

Similarly, B solves (**OP-B**):

$$\begin{aligned}
(\mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^B) &= \arg \max_{(\mathbf{q}^{B,B}, \mathbf{q}^{B,A}, \mathbf{Q}^B)} W_t^B(U^B(\mathbf{q}^B, \mathbf{Q}), U^A(\mathbf{q}^A, \mathbf{Q})) \\
\text{s.t. } &\mathbf{p}'_t(\mathbf{q}^A + \mathbf{q}^B) + \mathbf{P}'_t \mathbf{Q} \leq Y_t \\
&\mathbf{q}_t^{A,A} + \mathbf{q}_t^{B,A} = \mathbf{q}^A \\
&\mathbf{q}_t^{A,B} + \mathbf{q}_t^{B,B} = \mathbf{q}^B \\
&\mathbf{Q}_t^A + \mathbf{Q}^B = \mathbf{Q}
\end{aligned}$$

An allocation that solves both problems simultaneously is called a household equilibrium with caring.

Definition 1 *An allocation $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with caring if and only if it simultaneously solves **OP-A** and **OP-B**.*

Our new model enables us to define a measure of intrahousehold caring. To formalize this idea, let $\partial U^M(\mathbf{q}^M, \mathbf{Q})/\partial q_1$ represent the marginal utility of the numeraire (i.e. the first private good) for member $M \in \{A, B\}$ at the allocation $\{\mathbf{q}^M, \mathbf{Q}\}$. Then, for a public good k we define¹²

$$\tau_k^M(\mathbf{q}^M, \mathbf{Q}) \equiv \frac{\frac{\partial U^M}{\partial Q_k}}{\frac{\partial U^M}{\partial q_1^M}} \bigg|_{(\mathbf{q}^M, \mathbf{Q})}.$$

In words, the function value $\tau_k^M(\mathbf{q}^M, \mathbf{Q})$ gives member M 's marginal willingness to pay (MWTP) for an additional unit of k at $\{\mathbf{q}^M, \mathbf{Q}\}$.

¹²Throughout, we use $\frac{\partial U^M}{\partial q_n^M}$ for the partial derivative of the utility function U^M with respect to the consumption quantity of the private good n , and $\frac{\partial U^M}{\partial Q_k}$ for the partial derivative of the function U^M associated with the quantity of the public good k .

We can now derive the following result. (The proofs of our main results are given in Appendix A.)

Proposition 1 *Let $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ be a household equilibrium with caring. Then, there exist numbers $\theta_t^A, \theta_t^B \in [0, 1]$ such that for all public goods k :*

$$\max \{ \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t), \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) + \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) \} = P_{t,k}.$$

It follows from the proof of this proposition that the values of the indices θ_t^A and θ_t^B are determined by the curvatures of the caring functions W_t^A and W_t^B at equilibrium, which actually capture the degree of intrahousehold caring.¹³ Assumption SC guarantees that θ_t^A and θ_t^B are both contained in the unit interval. In the next section, we will use the dual representation of our consumption model to provide a specific equilibrium interpretation for the equality condition in Proposition 1.

To further enhance the intuition of our newly proposed model, we consider the two natural benchmark cases, i.e. the fully cooperative model and the noncooperative model without caring. In terms of Definition 1 (and problems **OP-A** and **OP-B**), if the caring functions W_t^A and W_t^B coincide (i.e. $W_t^A = W_t^B = W_t$), then both members optimize the same objective function. By construction, this implies a cooperative equilibrium (i.e. a Pareto optimal intrahousehold allocation). In this case, the caring function W_t corresponds to a so-called generalized (Samuelson) household welfare function (see, for example, Apps and Rees (2009)). By varying W_t , any Pareto efficient allocation can be reached as a household equilibrium with caring. By contrast, if the caring functions reduce to ‘egoistic’ functions (i.e. $W_t^A(U^A, U^B) = U^A$ and $W_t^B(U^B, U^A) = U^B$), then the household equilibrium reduces to a noncooperative equilibrium without caring. Our model is general in that it also captures all possible equilibrium situations between the fully cooperative equilibrium and the noncooperative equilibrium without caring.

Using the same two benchmark models, we can effectively interpret the indices θ_t^A and θ_t^B in Proposition 1 as capturing the degree of cooperation at the equilibrium intrahousehold allocation. First, in a cooperative equilibrium the MWTP functions τ_k^M coincide with the so-called Lindahl prices. In particular, it is well known that any Pareto efficient allocation $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t\}$ must satisfy the Lindahl-Bowen-Samuelson

¹³Formally, we have $\theta_t^A = \left(\frac{\partial W_t^B}{\partial U^A} / \frac{\partial W_t^B}{\partial U^B} \right) \left(\frac{\partial U^A}{\partial q_1^A} / \frac{\partial U^B}{\partial q_1^B} \right)$ and $\theta_t^B = \left(\frac{\partial W_t^A}{\partial U^B} / \frac{\partial W_t^A}{\partial U^A} \right) \left(\frac{\partial U^B}{\partial q_1^B} / \frac{\partial U^A}{\partial q_1^A} \right)$, where all partial derivatives are evaluated at the allocation $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t\}$. In words, θ_t^A equals the ratio of member B ’s marginal valuation for a unit increase of the numeraire quantity for member A (which enters the caring function W_t^B through U^A) relative to his marginal valuation for the same increase of the numeraire quantity for his own (which enters W_t^B through U^B). Likewise, the variable θ_t^B equals the ratio of A ’s marginal valuation for a unit increase of the numeraire quantity for B relative to her marginal valuation for the same quantity increase for her own.

conditions (see, for example, Samuelson (1954)). And, thus, we get for each public good k :

$$\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) = P_{t,k}.$$

In words, the sum of the members' MWTP must sum to the market prices. This case coincides with $\theta_t^A = \theta_t^B = 1$ in Proposition 1.

We next turn to the noncooperative model. In this case we get the following equilibrium condition for every public good k :

$$\max\{\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t), \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t)\} = P_{t,k};$$

see, for example, Cherchye, Demuynck and De Rock (2011). Thus, this case corresponds to $\theta_t^A = \theta_t^B = 0$ in Proposition 1.

More generally, if the indices θ_t^A and θ_t^B are closer to unity, the household will behave more as in the cooperative model. The duality result in Section 4 will provide an additional interpretation of θ_t^A and θ_t^B as quantifying the degree of intrahousehold cooperation of each member. In Section 5 we will show that it is possible to empirically recover the values of θ_t^A and θ_t^B . In this respect, we also note that $\max\{\theta_t^A, \theta_t^B\} < 1$ implies $\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) > P_{t,k}$ (because of Proposition 1), which reveals Pareto inefficient behavior. As such, θ_t^A and θ_t^B also indicate the extent of Pareto (in)efficiency at each decision situation t .

As a final remark, we note that the values of θ_t^A and θ_t^B are situation-dependent in the general version of our model. In practice, one may impose $\theta_t^A = \theta^A$ and $\theta_t^B = \theta^B$ for all t , which thus assumes a constant degree of intrahousehold cooperation over all decision situations. Again, this encompasses the fully cooperative model (with $\theta^A = \theta^B = 1$) and the noncooperative model without caring (with $\theta^A = \theta^B = 0$) as limiting cases. As a specific illustration, we will consider such constant intrahousehold cooperation in our empirical application in Section 5.

4 A duality result

The second fundamental theorem of welfare economics provides one of the most important theoretical insights related to the concept of Pareto efficiency. Specifically, provided that some regularity conditions are satisfied, any Pareto optimal allocation can be dually characterized in terms of a suitable income distribution and by making use of individual Lindahl prices for the publicly consumed goods (see, for example, Bergstrom (1976)). This dual characterization of Pareto optimality has often been used to provide a decentralized two-stage representation of the fully cooperative model of household consumption: in the first stage, the household divides the total income over the household members; in the second stage, each individual

member chooses a consumption allocation that maximizes her/his utility subject to the personalized budget constraint defined in the first stage.

In this section, we will develop a similar duality result for the noncooperative model with caring preferences that we introduced above: we will show that this model is dually equivalent to a noncooperative model with non-caring preferences that is characterized by intrahousehold transfers. The magnitude of these transfers will be directly related to the MWTP functions τ_k^A and τ_k^B and the indices θ_t^A and θ_t^B introduced in the previous section. In turn, this duality result implies a decentralized representation of the model that contains two stages. As we will explain, this representation will provide a further motivation to interpret θ_t^A and θ_t^B as measuring the degree of intrahousehold cooperation.

Before formally stating the duality result, we first explain the two stages of the noncooperative household model with transfers. In the first stage, the total household income Y_t is divided between A and B , which defines the individual incomes Y_t^A and Y_t^B (with $Y_t^A + Y_t^B = Y_t$). Here, we abstract from explicitly modeling this first step. Similar to our treatment of caring functions in the previous section, this intrahousehold income distribution can be seen as a function of situation-dependent exogenous variables (i.e. the so-called extra-environmental parameters or distribution factors). In the concluding section, we discuss the possibility to more carefully investigate this first step income distribution as an interesting avenue for follow-up research. At this point, we indicate that the idea of an intrahousehold income distribution resembles the so-called ‘sharing rule’ concept that applies to the fully cooperative model: in the decentralized representation of this model, the sharing equally defines the within-household income distribution underlying the (in casu Pareto efficient) household consumption decisions.¹⁴

In the second stage of the allocation process, each household member M ($= A$ or B) decides on the optimal level of her/his own private consumption and the own contribution to the level of public goods, by maximizing her/his own utility $U^M(\mathbf{q}^M, \mathbf{Q})$ subject to a personalized budget constraint defined by the individual income. In doing so, the individual faces the price vectors \mathbf{p}_t and \mathbf{P}_t for her/his choice of private consumption \mathbf{q}_t^M and public contribution \mathbf{Q}_t^M . In addition, each individual receives a transfer from the other individual per unit of public good that she/he purchases. We denote these transfers for each public good k by $\sigma_{t,k}^A$ and $\sigma_{t,k}^B$; $\boldsymbol{\sigma}_t^A$ and $\boldsymbol{\sigma}_t^B$ represent the corresponding vectors of intrahousehold transfers.

¹⁴In fact, Chiappori (1988, 1992) originally introduced this sharing rule concept for the model without public goods. In the literature on the cooperative model, a refinement of the concept that accounts for public goods is the so-called ‘conditional’ sharing rule. This concept captures how the group shares the income to be spent on private consumption for the given level of public consumption; see, for example, Blundell, Chiappori and Meghir (2005) for discussion. As such, this first step income distribution concept is not fully comparable to ours, which is not conditional on the level of public consumption.

There are at least two interpretations for these intrahousehold transfers related to public goods. First, one can see these transfers as voluntary contributions: as B benefits from the purchase of $Q_{t,k}^A$, it may be the case that she/he is willing to contribute to the purchase of this bundle. Next, one can also interpret them as representing an implicit tax that B has to pay for the benefit of receiving $Q_{t,k}^A$. Both interpretations express that intrahousehold transfers (i.e. a given specification of σ_t^A and σ_t^B) refer to the degree of (voluntary or obligatory) cooperation within the household.

Summarizing, at each decision situation t , member A faces the following dual optimization problem (**DOP-A**):

$$\begin{aligned} \{\mathbf{q}_t^A, \mathbf{Q}_t^A\} &\in \arg \max_{\mathbf{q}_t^A, \mathbf{Q}_t^A} U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B) \\ \text{s.t. } &\mathbf{p}_t' \mathbf{q}_t^A + (\mathbf{P}_t - \boldsymbol{\sigma}_t^B)' \mathbf{Q}_t^A + \boldsymbol{\sigma}_t^{A'} \mathbf{Q}_t^B \leq Y_t^A. \end{aligned}$$

Similarly, B solves (**DOP-B**):

$$\begin{aligned} \{\mathbf{q}_t^B, \mathbf{Q}_t^B\} &\in \arg \max_{\mathbf{q}_t^B, \mathbf{Q}_t^B} U^B(\mathbf{q}_t^B, \mathbf{Q}_t^B + \mathbf{Q}_t^A) \\ \text{s.t. } &\mathbf{p}_t' \mathbf{q}_t^B + (\mathbf{P}_t - \boldsymbol{\sigma}_t^A)' \mathbf{Q}_t^B + \boldsymbol{\sigma}_t^{B'} \mathbf{Q}_t^A \leq Y_t^B. \end{aligned}$$

It is easy to see that the two budget constraints add up to the household budget constraint at equilibrium (i.e. $\mathbf{p}_t' \mathbf{q}_t + \mathbf{P}_t' \mathbf{Q}_t \leq Y_t$).

Importantly, the noncooperative model under study does not explicitly consider caring preferences: in contrast to the model discussed in the previous section, the problems **DOP-A** and **DOP-B** do not include the caring functions W_t^A and W_t^B but only use the ‘egoistic’ functions U^A and U^B . However, as we will explain, our following concept of a household equilibrium with transfers accounts for caring preferences in an indirect way.

Definition 2 *An allocation $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with transfers if and only if it simultaneously solves **DOP-A** and **DOP-B** and, in addition, there exist θ_t^A and θ_t^B such that for all public goods k :*

$$\sigma_{t,k}^A = \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) \quad \text{and} \quad \sigma_{t,k}^B = \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t).$$

In this definition, an equilibrium household allocation requires that each member M ’s intrahousehold transfer related to public good k ($\sigma_{t,k}^M$) is proportional to M ’s MWTP for k ($\tau_k^M(\mathbf{q}_t^M, \mathbf{Q}_t)$). The factor of proportionality is giving by the index θ_t^M . Definition 2 establishes a direct link between the noncooperative model with caring introduced in the previous section (with problems **OP-A** and **OP-B**) and the two-stage allocation process discussed here (with problems **DOP-A** and **DOP-B**).

In the previous section, we argued that the curvatures of the caring functions W_t^A and W_t^B define θ_t^A and θ_t^B . As such, the condition on the intrahousehold transfers in Definition 2 indirectly incorporates caring preferences in the household equilibrium under consideration.

Interestingly, Definition 2 provides an additional interpretation of each index θ_t^M in terms of intrahousehold cooperation. Given member M 's MWTP for the public good k ($\tau_k^M(\mathbf{q}_t^M, \mathbf{Q}_t)$), θ_t^M captures the transfer M is willing to give to the other member L ($L \neq M$) if L purchases an additional unit of good k . In the fully cooperative case, M is willing to donate the full amount $\tau_k^M(\mathbf{q}_t^B, \mathbf{Q}_t)$ to L , which means $\theta_t^M = 1$. In this case, Definition 2 coincides with the standard definition of a Lindahl equilibrium. By contrast, in the noncooperative case without caring, M will not donate anything to L , so that $\theta_t^M = 0$. Now, Definition 2 reduces to the usual definition of a noncooperative equilibrium without caring. Apart from these fully cooperative and noncooperative cases, Definition 2 also includes the intermediate case in which M picks a number θ_t^M between 0 and 1 such that she/he donates a fraction θ_t^M of $\tau_k^M(\mathbf{q}_t^M, \mathbf{Q}_t)$ to L . Generally, a higher (lower) θ_t^M means that M is willing to cooperate more (less) with L .

Using Definition 2, we get the following first order conditions for **DOP-A** and **DOP-B** with respect to the public good k :

$$\max \{ \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t); \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) + \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) \} = P_{t,k}.$$

This condition is identical to the equilibrium condition in Proposition 1. However, the underlying interpretation is different, because we now start from the optimization problems **DOP-A** and **DOP-B** rather than **OP-A** and **OP-B**.

By considering θ_t^A and θ_t^B as capturing intrahousehold transfers, we can provide an intuitive equilibrium interpretation to the above equality condition. To see this, let us consider the two possible inequality situations. First, if $\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) > P_{t,k}$ then the total amount that A is willing to spend for an additional unit of public good k (i.e. A 's MWTP plus the fraction θ_t^B of B 's MWTP) exceeds the price A has to pay (i.e. $P_{t,k}$). In this case, A will effectively increase her holdings of good k . A directly analogous interpretation applies to the situation $\tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) + \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) > P_{t,k}$. And, thus, $\max \{ \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t); \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) + \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) \} > P_{t,k}$ implies a disequilibrium. Similarly, if we have $\max \{ \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t); \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) + \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) \} < P_{t,k}$, then either A or B (whoever contributes positively to good k) will want to decrease her/his contribution to k . Again, this implies a disequilibrium situation.

We are now in a position to establish the dual equivalence result mentioned above. Specifically, the following proposition implies that the household model with caring and the household model with transfers are empirically indistinguishable.

Proposition 2 *Let U^A and U^B be a pair of utility functions. Then, the following holds for any decision situation t :*

1. *Suppose $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with caring.*

Then, there exist individual incomes Y_t^A and Y_t^B (with $Y_t^A + Y_t^B = Y_t$) and indices θ_t^A and θ_t^B such that $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with transfers.

2. *Suppose $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with transfers.*

Then, there exist caring functions W_t^A and W_t^B and bundles $\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B$ (with $\mathbf{q}_t^A = \mathbf{q}_t^{A,A} + \mathbf{q}_t^{A,B}$, $\mathbf{q}_t^B = \mathbf{q}_t^{B,A} + \mathbf{q}_t^{B,B}$ and $\mathbf{Q}_t = \mathbf{Q}_t^A + \mathbf{Q}_t^B$) such that $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with caring.

5 Testable implications

So far, we have focused on the theoretical properties of our household model with caring (or, equivalently, with transfers). In this section, we show that the model has useful testable implications for empirical data. Specifically, we will focus on testable conditions in terms of revealed preferences. As indicated in the introduction, this revealed preference approach has been successfully applied for empirical analysis of non-unitary consumption models. In addition, recent methodological advances of Blundell, Browning and Crawford (2003, 2008) greatly enhanced the empirical usefulness of this revealed preference approach.

In the household consumption literature, empirical studies usually build on a differential characterization (rather than a revealed preference characterization) of household consumption models. The specific feature of this differential approach is that it focuses on properties of functions representing household consumption behavior (e.g. cost, indirect utility and demand functions),¹⁵ whereas the revealed preference approach (only) uses a finite set of household consumption observations. In this respect, Cherchye, De Rock and Demuynck (2011) point out that the revealed preference approach has some attractive features as compared to the more common differential approach for analyzing non-unitary consumption behavior. Most notably,

¹⁵The term ‘differential’ refers to the fact that the characterization is obtained by integrating and/or differentiating the functional specifications of the fundamentals of the model (e.g. the individual preferences of the household members). For differential characterizations of non-unitary consumption models, see Browning and Chiappori (1998) and Chiappori and Ekeland (2006, 2009), who focused on the cooperative model, and Lechene and Preston (2005, 2008), who considered the noncooperative model without caring.

contrary to existing results for the differential approach, the revealed preference characterization of the noncooperative model (without caring) is independent from (or non-nested with) the characterization of the cooperative model: a set of observations that satisfies the cooperative conditions does not necessarily satisfy the noncooperative conditions, and vice versa. More generally, this implies that models characterized by different degrees of intrahousehold cooperation (or caring) are independent of each other in terms of their revealed preference characterization. Clearly, this independence makes it interesting to compare the empirical validity of the different models. This is particularly relevant in the present context, as our empirical exercise will carry out such a comparison.

In our empirical exercise, we will apply the revealed preference conditions to analyze data taken from the Russia Longitudinal Monitoring Survey (RLMS). This application will demonstrate the practical relevance of the household model with caring. In addition, it will show that the revealed preference conditions allow us to recover the indices θ_t^A and θ_t^B , which -to recall- capture the degree of intrahousehold cooperation for behavior that is consistent with the model.

Revealed preference characterization: We start from a finite set of $|T|$ observed decision situations (or ‘observations’), i.e. $S = \{\mathbf{p}_t, \mathbf{P}_t; \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$. We remark that this implies minimal conditions on what is observed. In particular, we assume that at each observation t we only observe the price vectors \mathbf{p}_t and \mathbf{P}_t and the household consumption bundles \mathbf{q}_t and \mathbf{Q}_t .

Given our discussion in the previous sections, we consider the following definition of rationalizability.

Definition 3 Consider a data set $S = \{\mathbf{p}_t, \mathbf{P}_t; \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$. We say that S is rationalizable with caring if there exist utility functions U^A and U^B and, for each decision situation t , there exist caring functions W_t^A and W_t^B and bundles $\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B$ (with $\mathbf{q}_t^A = \mathbf{q}_t^{A,A} + \mathbf{q}_t^{A,B}$, $\mathbf{q}_t^B = \mathbf{q}_t^{B,A} + \mathbf{q}_t^{B,B}$ and $\mathbf{Q}_t = \mathbf{Q}_t^A + \mathbf{Q}_t^B$) such that $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with caring.

Before providing testable revealed preference conditions for rationalizability, we briefly recapture a result of Varian (1982; based on Afriat, 1967). Consider a finite set of $|L|$ observations, i.e. a set $Z = \{\mathbf{w}_l; \mathbf{x}_l\}_{l \in L}$ containing price vectors \mathbf{w}_l and quantity vectors \mathbf{x}_l . Then, we say that this set Z can be rationalized by a utility function U if each quantity bundle \mathbf{x}_l maximizes the function U in the following sense:

$$\mathbf{x}_l \in \arg \max_{\mathbf{x}} U(\mathbf{x}) \text{ s.t. } \mathbf{w}_l' \mathbf{x} \leq \mathbf{w}_l' \mathbf{x}_l.$$

Varian (1982) has shown that such a rationalizing utility function U exists if and only if the set Z satisfies the Generalized Axiom of Revealed Preference (GARP).

Definition 4 Consider a set $Z = \{\mathbf{w}_l; \mathbf{x}_l\}_{l \in L}$. For any $l_1, l_2 \in L$, $\mathbf{x}_{l_1} R^D \mathbf{x}_{l_2}$ if $\mathbf{w}_{l_1} \mathbf{x}_{l_1} \geq \mathbf{w}_{l_1} \mathbf{x}_{l_2}$. Next, $\mathbf{x}_{l_1} R^D \mathbf{x}_{l_2}$ if there exist a sequence r, \dots, t (with $r, \dots, t \in L$) such that $\mathbf{x}_{l_1} R^D \mathbf{x}_r, \dots, \mathbf{x}_t R^D \mathbf{x}_{l_2}$. The set Z satisfies GARP if, for all $l_1, l_2 \in L$, $\mathbf{x}_{l_1} R^D \mathbf{x}_{l_2}$ implies $\mathbf{w}_{l_2} \mathbf{x}_{l_1} \geq \mathbf{w}_{l_2} \mathbf{x}_{l_2}$.

Using Definition 4, we can characterize a data set S that is rationalizable with caring.

Proposition 3 Consider a data set $S = \{\mathbf{p}_t, \mathbf{P}_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$. The following conditions are equivalent:

1. The data set $S = \{\mathbf{p}_t, \mathbf{P}_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$ is rationalizable with caring.
2. For all decision situations t and public goods k there exist indices $\theta_t^A, \theta_t^B \in [0, 1]$, vectors $\boldsymbol{\tau}_t^A = (\tau_{t,1}^A, \dots, \tau_{t,K}^A)$, $\boldsymbol{\tau}_t^B = (\tau_{t,1}^B, \dots, \tau_{t,K}^B) \in \mathbb{R}_+^K$, and bundles $\mathbf{q}_t^A, \mathbf{q}_t^B \in \mathbb{R}_+^N$ such that

$$\mathbf{q}_t^A + \mathbf{q}_t^B = \mathbf{q}_t, \quad (\text{S.1})$$

$$\max \{ \tau_{t,k}^A + \theta_t^B \tau_{t,k}^B, \tau_{t,k}^B + \theta_t^A \tau_{t,k}^A \} = P_{t,k}, \text{ and} \quad (\text{S.2})$$

$$\{\mathbf{p}_t, \boldsymbol{\tau}_t^A; \mathbf{q}_t^A, \mathbf{Q}_t\}_{t \in T} \text{ and } \{\mathbf{p}_t, \boldsymbol{\tau}_t^B; \mathbf{q}_t^B, \mathbf{Q}_t\}_{t \in T} \text{ satisfy GARP.} \quad (\text{S.3})$$

Moreover, it follows that there exists $\mathbf{Q}_t^A, \mathbf{Q}_t^B \in \mathbb{R}_+^K$ such that

$$\text{if } \tau_{t,k}^A + \theta_t^B \tau_{t,k}^B < P_{t,k} \text{ then } Q_{t,k}^A = 0 \text{ and } Q_{t,k}^B = Q_{t,k}, \text{ and} \quad (\text{S.4})$$

$$\text{if } \theta_t^A \tau_{t,k}^A + \tau_{t,k}^B < P_{t,k} \text{ then } Q_{t,k}^B = 0 \text{ and } Q_{t,k}^A = Q_{t,k}. \quad (\text{S.5})$$

The explanation is as follows. The restriction S.1 requires the individual consumption bundles for the private goods to sum to the demanded household bundle of private goods. The restriction S.2 corresponds to the equilibrium condition for the public goods k in Proposition 1 (for a positive consumption of the public good k). Condition S.3 states that rationalizability implies a GARP condition at the level of individuals A and B , which corresponds to the existence of the individual utility functions U^A and U^B in Definition 3. The specificity of our model is that these GARP conditions use MWTP vectors ($\boldsymbol{\tau}_t^A$ and $\boldsymbol{\tau}_t^B$) for evaluating the publicly consumed quantities (\mathbf{Q}_t). Finally, the conditions S.4 and S.5 follow from the fact that, if $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B < P_{t,k}$ ($\theta_t^A \tau_{t,k}^A + \tau_{t,k}^B < P_{t,k}$), then A (B) will sell back any positive amount of the public good k . This implies $Q_{t,k}^A = 0$ ($Q_{t,k}^B = 0$) and, thus, $Q_{t,k}^B = Q_{t,k}$ ($Q_{t,k}^A = Q_{t,k}$).

Testing and recovery: In Appendix B, we show that the revealed preference conditions in Proposition 3 can be reformulated in mixed integer programming (MIP) terms. This complements existing MIP characterizations of the cooperative model (in Cherchye, De Rock and Vermeulen (2011)) and the noncooperative model without caring (in Cherchye, Demuynck and De Rock (2011)). The attractive feature of the MIP characterization is that it allows for checking consistency of a given data S with the conditions in Proposition 3. In the spirit of Varian (1982), we refer to this as ‘testing’ data consistency with the model under study.¹⁶

More specifically, we demonstrate in Appendix B that all constraints of the MIP formulation are linear for fixed θ_t^A and θ_t^B . Linearity implies that the above program can be solved by standard MIP methods for a given data set S . If we do not know the values of θ_t^A and θ_t^B (which is usually the case), then we suggest to conduct a grid search that checks the above problem (through MIP methods) for a whole range of possible values for θ_t^A and θ_t^B . In our empirical application, we will assume constant θ_t^A and θ_t^B , i.e. $\theta_t^A = \theta^A$ and $\theta_t^B = \theta^B$ for all t ; and we will use an equally sparse grid search with step 0.1 for $\theta^A, \theta^B \in [0, 1]$, which implies that we consider 121 different combinations of θ^A and θ^B . The fact that the parameters θ^A and θ^B are independent of t will simplify our presentation of the empirical results. The underlying assumption is that the degree of intrahousehold cooperation does not change over the observations. It is possible to relax this assumption, but this would come at the cost of a considerable increase of the computational complexity of the testable MIP conditions.

If observed behavior is consistent with our model with caring (i.e. the set S is rationalizable with caring), then a natural next question pertains to recovering/identifying structural features of the decision model that underlies the (rationalizable) observed consumption behavior. In our application, we will illustrate recovery/identification of values for θ^A and θ^B (assuming $\theta_t^A = \theta^A$ and $\theta_t^B = \theta^B$ for all t ; see above) that are consistent with a rationalization of a given set S . Given our discussion in the preceding sections, these values can be interpreted in terms of intrahousehold cooperation (or caring) that is revealed in the observed consumption behavior. Other recovery questions may pertain to the MWTP values $\tau_{t,k}^M(\mathbf{q}^M, \mathbf{Q})$ and individual income shares Y_t^M at equilibrium (in terms of the household model with transfers; see Definition 2). Generally, such recovery can start from the MIP methodology presented in this paper. In this respect, we can refer to Cherchye, De Rock and Vermeulen (2011), who consider these questions for the cooperative model; their analysis is directly extended to the noncooperative model with caring discussed here. These authors’ basic argument is that revealed preference recovery on the basis

¹⁶As is standard in the revealed preference literature, the type of tests that we consider here are ‘sharp’ tests; either a data set satisfies the data consistency conditions or it does not.

of an MIP characterization of rational behavior boils down to defining feasible sets characterized by the MIP constraints.

As for recovery of the individual income shares, one important final remark pertains to restrictions S.4 and S.5 in Proposition 3. As we will explain below, these restrictions imply that the shares Y_t^A and Y_t^B that underlie observed (rationalizable) behavior are not identifiable in general. This contrasts with the cooperative case in which the within-household income distribution (in general) can be identified from the observed set S . This identifiability result does not generally hold under noncooperative behavior with caring. As a matter of fact, this identifiability problem for our model actually parallels a similar problem for the noncooperative model without caring.¹⁷

To see the identifiability problem, we first note that the budget constraints in **DOP-A** and **DOP-B** imply

$$\begin{aligned} \mathbf{p}'_t \mathbf{q}_t^A + (\mathbf{P}_t - \theta_t^B \boldsymbol{\tau}_t^B)' \mathbf{Q}_t^A + \theta_t^A \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B &= Y_t^A \text{ and} \\ \mathbf{p}'_t \mathbf{q}_t^B + (\mathbf{P}_t - \theta_t^A \boldsymbol{\tau}_t^A)' \mathbf{Q}_t^B + \theta_t^B \boldsymbol{\tau}_t^{B'} \mathbf{Q}_t^A &= Y_t^B. \end{aligned}$$

Thus, because of conditions S.4 and S.5 we obtain that Y_t^A and Y_t^B are uniquely identified only if for all k and t we have $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B < P_{t,k}$ (so that $Q_{t,k}^A = 0$ and $Q_{t,k}^B = Q_{t,k}$) or $\tau_{t,k}^B + \theta_t^A \tau_{t,k}^A < P_{t,k}$ (so that $Q_{t,k}^B = 0$ and $Q_{t,k}^A = Q_{t,k}$). In terms of the noncooperative model without caring, this last situation would conform to the so-called separate spheres concept.¹⁸

On the other hand, as soon as there is one public good k to which both individuals contribute for some t (i.e. $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B = \tau_{t,k}^B + \theta_t^A \tau_{t,k}^A = P_{t,k}$), it is impossible to exactly recover the income shares Y_t^A and Y_t^B are consistent with a rationalization of the given data. Specifically, in this case $Q_{t,k}^A$ and $Q_{t,k}^B$ can take any value (under the sole condition $Q_{t,k}^A + Q_{t,k}^B = Q_{t,k}$) and, thus, the expenditures on good k cannot be assigned to the individual household members. Interestingly, this last result complies with the so-called local income pooling result for the noncooperative model without caring.¹⁹

Empirical Illustration: To demonstrate the practical usefulness of the revealed preference conditions in Proposition 3, we provide a brief empirical illustration.

¹⁷See Cherchye, Demuyne and De Rock (2011) for more discussion on the identification of individual income shares on the basis of testable revealed preference conditions for the noncooperative model without caring.

¹⁸See, for example, Lundberg and Pollak (1993) and Browning, Chiappori and Lechene (2010).

¹⁹See, for example, Kemp (1984), Bergstrom, Blume and Varian (1986) and Browning, Chiappori and Lechene (2010). Importantly, even though we cannot identify Y_t^A and Y_t^B under jointly contributed public goods, it is still possible to recover upper and lower bounds on values for Y_t^A and Y_t^B that are consistent with a rationalization with caring of the given data set. These bounds then account for the total (non-assignable) expenditures on the jointly contributed public goods.

Specifically, we consider an application of our rationalizability restrictions to data taken from the Russia Longitudinal Monitoring Survey (RLMS). Cherchye, De Rock and Vermeulen (2009, 2011) conducted a revealed preference analysis of these data in terms of the cooperative model, while Cherchye, Demuynck and De Rock (2011) analyzed consistency with the noncooperative model without caring. For compactness, we refer to these authors for a detailed discussion of the data. We extend these earlier studies by analyzing the same data in terms of our noncooperative model with caring.

The data set consists of 148 adult couples, with both (female and male) household members employed. For each separate household, the data set has 8 ($= |T|$) observations (prices and quantities) on 21 nondurable goods: 3 public goods and 18 private goods ($K = 3$ and $N = 18$).²⁰ Each household is considered separately, which avoids (debatable) preference homogeneity assumptions across males or females of different households. As such, the degree of cooperation may vary for different households. As for each individual household, we assume the degree of intrahousehold cooperation is constant over all observed decision situations, i.e. we consider $\theta_t^A = \theta^A$ and $\theta_t^B = \theta^B$ for all t . This considerably facilitates our following discussion. In this respect, we recall that the fully cooperative model and the noncooperative model without caring correspond to $\theta^A = \theta^B = 1$ and $\theta^A = \theta^B = 0$, respectively.

To focus our discussion, we directly build on an empirical finding of Cherchye, Demuynck and De Rock (2011). Starting from the original sample with 148 households, these authors identified two households (1 and 2) that seem particularly interesting to illustrate the empirical usefulness of our newly proposed model: household 1 can be rationalized in terms of the noncooperative model but not in terms of the cooperative model, and household 2 can be rationalized in terms of the cooperative model but not in terms of the noncooperative model. For ease of exposition, we will only report rationalizability results for these two households.²¹

Table 1 gives results for our (MIP) rationalizability test (1 = pass; 0 = fail) that correspond to 121 combinations of $\theta^A, \theta^B \in [0, 1]$. Interestingly, these results suggest that revealed preference conditions can be useful to identify values for θ^A and θ^B that are consistent with rationalizable behavior. Given that θ^A and θ^B indicate the degree of cooperation of each individual household member, these values tell us about

²⁰The public goods are (1) wood fuel, (2) gas fuel and (3) housing rent. The private goods are (1) food outside the home, (2) clothing, (3) car fuel, (4) luxury goods, (5) services, (6) bread, (7) potatoes, (8) vegetables, (9) fruit, (10) meat, (11) dairy products, (12) fat, (13) sugar, (14) eggs, (15) fish, (16) other food items, (17) alcohol and (18) tobacco. Following Cherchye, De Rock and Vermeulen (2011), we use assignable information for the private goods. Specifically, our following results pertain to fully observing the private quantities \mathbf{q}_t^A and \mathbf{q}_t^B in Proposition 3. Generally, using assignable information enhances the power of the empirical analysis. However, strictly speaking it is not needed for such empirical analysis to be possible (e.g., the testable conditions in Proposition 3 do not require such information).

²¹Results for other households are available upon request.

the extent to which observed household consumption behavior is characterized by (limited) intrahousehold cooperation. For example, for household 1 consistency with the rationalizability conditions in Proposition 3 holds if $\theta^A, \theta^B \leq 0.6$ (conditional on the grid search that we conducted). Similarly, for household 2 rationalizability holds if $\theta^A \geq 0.6$ and $\theta^B \geq 0.4$.

In fact, these results show that the degree of cooperation in the household equilibrium may vary across households (e.g. households 1 and 2 are characterized by different values for θ^A and θ^B) and household members (e.g. θ^A and θ^B have different values for household 2). In our opinion, an interesting following step can relate these findings on (varying) intrahousehold cooperation to specific characteristics of the household and/or household members. Such an exercise falls beyond the scope of the current study (also because of limited data availability). But the results in Table 1 clearly suggest that our model with caring (and the corresponding revealed preference conditions) provides a useful theoretical basis for empirically addressing this type of questions.

Apart from test results, Table 1 also provides power estimates for the two households and the rationalizability tests that we consider (corresponding to different combinations of θ^A and θ^B). Indeed, discriminatory power is often conceived as an important consideration to evaluate a particular behavioral model, and to compare different models, in terms of practical usefulness. In our specific context, it therefore seems interesting to compare the power of consumption models characterized by different degrees of intrahousehold cooperation.

For a given data set, power quantifies the probability of detecting (simulated) behavior that is not consistent with the behavioral model subject to testing; we will refer to such inconsistent behavior as ‘random’ behavior. In our application, we simulate random behavior by using a bootstrap method.²² For each household, we simulate 1000 random series of eight consumption choices by constructing, for each of the eight observed household budgets, a random quantity bundle exhausting the given budget (for the corresponding prices); we construct these random quantity bundles by drawing budget shares (for the 21 goods) from the set of 1184 (= 148 x 8) observed household choices in the original data set. The power measure is then calculated as one minus the proportion of the randomly generated consumption series that are consistent with the model under evaluation. By using this bootstrap method, our power assessment gives information on the expected distribution of violations under random choice, while incorporating information on the households’ actual choices.

From Table 1, we learn that, for both households under study, the power is

²²See Bronars (1987), Andreoni and Harbaugh (2006) and Beatty and Crawford (2010) for general discussions on alternative procedures to evaluate power in the context of revealed preference tests such as ours.

about the same for the different combinations of θ^A and θ^B that we consider. In fact, the power of ‘intermediate’ models with $\theta^A, \theta^B \in [0, 1]$ is generally closely similar to the power of the ‘extreme’ fully cooperative and noncooperative models (with, respectively, $\theta^A = \theta^B = 1$ and $\theta^A = \theta^B = 0$). In our opinion, this provides an empirical motivation for considering non-unitary models with various degrees of cooperation when analyzing household consumption behavior. In some cases, a household model with limited intrahousehold cooperation may provide a better description of the actual household consumption behavior than models with full or without any cooperation.

[Table 1 about here]

6 Conclusion

We have presented a model for analyzing household consumption behavior that simultaneously accounts for caring preferences and noncooperative behavior in decisions on public goods. Interestingly, by varying the degree of intrahousehold caring, the model encompasses a whole continuum of household consumption models situated between the fully cooperative model and the noncooperative model without caring. Attractively, our newly proposed model also allowed us to define a measure for the degree of intrahousehold cooperation. Following a revealed preference approach, we derived the testable implications of the model for empirical data. We have illustrated our theoretical discussion through an empirical application to RLMS data. This application suggested the empirical relevance of considering a noncooperative model with caring in addition to the fully cooperative model and the noncooperative model without caring. In addition, it demonstrated the possibility to empirically recover our measure for the degree of cooperation within a particular household.

We see at least three interesting directions for follow-up research. First, in the (two-stage) dual representation of our model as characterized by intrahousehold transfers (see Section 4), we have taken the (first stage) intrahousehold income distribution as exogenously given. In this respect, we recall that the methodology presented in Section 5 effectively allows for recovering the income distribution associated with observed household behavior that is found to be consistent with our model. A natural following step of the analysis may relate this income distribution to different (household or member specific) factors that impact on it. In fact, such research would be similar in spirit to existing research focusing on ‘distribution factors’ in the context of the cooperative model of household consumption. See, for example, Bourguignon, Browning and Chiappori (2009) for a recent discussion of

testable implications (for this cooperative model) that are induced by distribution factors.

Second, in Section 5 we have adopted a revealed preference approach to establish the testable implications of the newly proposed model. Because this revealed preference approach directly applies the theoretical implications of our model to the observed consumption behavior, we believe it is natural to adopt this approach as a first assessment tool for the empirical applicability of our newly proposed consumption model. In addition, as we have discussed, this revealed preference approach has proven to be particularly successful for empirical analysis of non-unitary consumption models. Furthermore, we have argued that the approach has a number of attractive features as compared to the more traditional differential approach to characterizing non-unitary consumption models. However, we also believe that an interesting extension of the results in this paper consists of developing the differential counterparts of the conditions presented in Section 5. Such an extension would complement existing results for the cooperative model (see Browning and Chiappori (1998), Chiappori and Ekeland (2006, 2009) and Donni (2009)) and the noncooperative model without caring (see Lechene and Preston (2005, 2008)). In this respect, a fruitful starting point may be the study of d’Aspremont and Dos Santos Ferreira (2009), who consider a differential characterization of an alternative model that is situated between the fully cooperative and noncooperative models.²³

Finally, we have considered a static framework, and abstracted from dynamic (or intertemporal) considerations in household consumption behavior. Clearly, developing a static model provides a logical first step towards defining a dynamic model. For example, if one assumes intertemporal separability of consumption decisions, then data consistency with the static model is a necessary condition for data consistency with any dynamic model. As for establishing a dynamic model of noncooperative household consumption with caring preferences, one may usefully combine the insights of this paper with the approach developed in Mazzocco (2007), who focused on fully cooperative household behavior. As for establishing the associated revealed preference testable conditions, one may fruitfully build on the analysis in Browning (1989) and Crawford (2010), who considered intertemporal consumption behavior in a unitary framework.

²³In this respect, an important difference between our model and the model of d’Aspremont and Dos Santos Ferreira is that our model uses information (e.g. MWTP) for quantities that are effectively observed (i.e. the equilibrium bundles), while the alternative model of d’Aspremont and Dos Santos Ferreira requires information for quantities in some unobserved cooperative equilibrium (associated with the same observation, i.e. prices and income). For example, the fact that our model only uses observable quantity information allowed us to reformulate the revealed preference characterization in Proposition 3 in MIP terms. As far as we can see, it is not possible to obtain a similar MIP formulation for the revealed preference characterization of the model of d’Aspremont and Dos Santos Ferreira, precisely because this model requires unobservable quantity information.

Appendix A: proofs

Proof of Proposition 1

The first order conditions for **OP-A** and **OP-B** with respect to the numeraire (i.e. the first private good) and public goods k are

$$\frac{\partial W_t^A}{\partial U^A} \frac{\partial U^A}{\partial q_1^A} \leq \lambda_t^A, \quad (1)$$

$$\frac{\partial W_t^B}{\partial U^A} \frac{\partial U^A}{\partial q_1^A} \leq \lambda_t^B, \quad (2)$$

$$\frac{\partial W_t^A}{\partial U^B} \frac{\partial U^B}{\partial q_1^B} \leq \lambda_t^A, \quad (3)$$

$$\frac{\partial W_t^B}{\partial U^B} \frac{\partial U^B}{\partial q_1^B} \leq \lambda_t^B, \quad (4)$$

$$\frac{\partial W_t^A}{\partial U^A} \frac{\partial U^A}{\partial Q_k} + \frac{\partial W_t^A}{\partial U^B} \frac{\partial U^B}{\partial Q_k} \leq \lambda_t^A P_{t,k}, \quad (5)$$

$$\frac{\partial W_t^B}{\partial U^A} \frac{\partial U^A}{\partial Q_k} + \frac{\partial W_t^B}{\partial U^B} \frac{\partial U^B}{\partial Q_k} \leq \lambda_t^B P_{t,k}, \quad (6)$$

with λ_t^A and λ_t^B the Lagrange multipliers of the respective budget constraints. We start from the following observations:

- Either (1) or (2) must hold with equality. This follows from the fact that $q_{t,1}^A$ is strictly positive.
- Either (3) or (4) must hold with equality. This follows from the fact that $q_{t,1}^B$ is strictly positive.
- Either (5) or (6) must hold with equality. This follows from the fact that $Q_{t,k}$ is strictly positive.
- Not both (1) and (4) have strict inequality.

Proof. We prove ad absurdum. Suppose both (1) and (4) hold with strict inequality, then by the first two observations above, it must be that (2) and (3) hold with equality. Then, dividing condition (1) by (2) gives:

$$\frac{\frac{\partial W_t^A}{\partial U^A}}{\frac{\partial W_t^B}{\partial U^A}} < \frac{\lambda_t^A}{\lambda_t^B},$$

while dividing (3) by (4) gives:

$$\frac{\lambda_t^A}{\lambda_t^B} < \frac{\frac{\partial W_t^A}{\partial U^B}}{\frac{\partial W_t^B}{\partial U^B}}.$$

These two inequalities impose that:

$$-\frac{\frac{\partial W_t^A}{\partial U^A}}{\frac{\partial W_t^A}{\partial U^B}} > -\frac{\frac{\partial W_t^B}{\partial U^A}}{\frac{\partial W_t^B}{\partial U^B}}.$$

This contradicts Assumption SC. ■

The above reasoning gives us three possible cases: (i) both (1) and (3) hold with equality, (ii) both (1) and (4) hold with equality, (iii) both (2) and (4) hold with equality.

Case (i) In this case, equation (5) can be rewritten as

$$(\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t)) \leq P_{t,k} \quad (7)$$

Further, we have that,

$$\frac{\partial W_t^B}{\partial U^A} \frac{\partial U^A}{\partial Q_k} + \frac{\partial W_t^B}{\partial U^B} \frac{\partial U^B}{\partial Q_k} \leq \lambda_t^B (\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t)) \quad (8)$$

$$\leq \lambda_t^B P_{t,k} \quad (9)$$

The inequality in (8) follows from using conditions (2) and (4). The inequality in (9) follows from (7).

As one of the two conditions (5) or (6) must hold with equality, we have that that $\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) = P_{t,k}$. As k was arbitrary, this holds for every public good. Setting $\theta_t^A = \theta_t^B = 1$ demonstrates the proof.

Case (ii) For this case, we can rewrite conditions (5) and (6) as:

$$\begin{aligned} \frac{\partial W_t^A}{\partial U^A} \frac{\partial U^A}{\partial Q_k} + \frac{\partial W_t^A}{\partial U^B} \frac{\partial U^B}{\partial Q_k} &= \lambda_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \frac{\frac{\partial W_t^A}{\partial U^B}}{\frac{\partial W_t^B}{\partial U^B}} \lambda_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) \\ &\leq \lambda_t^A P_{t,k} \end{aligned}$$

and,

$$\begin{aligned} \frac{\partial W_t^B}{\partial U^A} \frac{\partial U^A}{\partial Q_k} + \frac{\partial W_t^B}{\partial U^B} \frac{\partial U^B}{\partial Q_k} &= \frac{\frac{\partial W_t^B}{\partial U^A}}{\frac{\partial W_t^A}{\partial U^A}} \lambda_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \lambda_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) \\ &\leq \lambda_t^B P_{t,k} \end{aligned}$$

As one of these two conditions must hold with equality, we have that:

$$\max\{\tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t), \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) + \theta_t^A \tau_k^A(\mathbf{q}_t^A, \mathbf{Q}_t)\} = P_{t,k} \quad (10)$$

where

$$\theta_t^A = \frac{\frac{\partial W_t^B}{\partial U^A} \lambda_t^A}{\frac{\partial W_t^A}{\partial U^A} \lambda_t^B} \leq \frac{\lambda_t^B \lambda_t^A}{\lambda_t^A \lambda_t^B} = 1 \quad (11)$$

and,

$$\theta_t^B = \frac{\frac{\partial W_t^A}{\partial U^B} \lambda_t^B}{\frac{\partial W_t^B}{\partial U^B} \lambda_t^A} \leq \frac{\lambda_t^A \lambda_t^B}{\lambda_t^B \lambda_t^A} = 1 \quad (12)$$

The inequality in (11) follows from dividing condition (2) by (1) while the inequality in (12) follows from dividing condition (3) by (4).

Case(iii) This case is analogous to case (i) and is left to the reader.

Proof of Proposition 2

Proof of statement 1

Assume that for each decision situation t we have that $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ satisfies the definition of a household equilibrium with caring for the utility functions U^A, U^B , caring functions W_t^A, W_t^B , prices $\mathbf{p}_t, \mathbf{P}_t$ and household income Y_t .

We need to show that there exist numbers $\theta_t^A, \theta_t^B \in [0, 1]$ and incomes Y_t^A, Y_t^B (with $Y_t^A + Y_t^B = Y_t$) such that $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with transfers. Let us first focus on individual A . For the proof, we will again distinguish three cases, identical to the cases used in the proof of Proposition 1.

Before we begin, consider the first order condition for A and B with respect to

the n th private good for A (i.e. the quantities $q_{t,n}^{A,A}$ and $q_{t,n}^{B,A}$):

$$\frac{\partial W_t^A}{\partial U^A} \frac{\partial U^A}{\partial q_n^A} \leq \lambda_t^A p_{t,n} \quad (13)$$

$$\frac{\partial W_t^B}{\partial U^A} \frac{\partial U^A}{\partial q_n^A} \leq \lambda_t^B p_{t,n} \quad (14)$$

Lemma 1 *If case (i) or (ii) holds and $q_{t,n}^A > 0$, then (13) holds with equality for all private goods s at equilibrium. On the other hand if case (iii) holds and $q_{t,n}^A > 0$, then (14) holds with equality for all private goods n at equilibrium.*

Proof. Assume that either case (i) or (ii) holds and that $\frac{\partial W_t^A}{\partial U^A} \frac{\partial U^A}{\partial q_n^A} < \lambda_t^A p_{t,n}$. Then, since $q_{t,n}^A > 0$ it must be that $\frac{\partial W_t^B}{\partial U^A} \frac{\partial U^A}{\partial q_n^A} = \lambda_t^B p_{t,n}$. Dividing these two conditions gives:

$$\begin{aligned} \frac{\frac{\partial W_t^A}{\partial U^A}}{\frac{\partial W_t^B}{\partial U^A}} &< \frac{\lambda_t^A}{\lambda_t^B}. \\ \frac{\frac{\partial W_t^A}{\partial U^A}}{\frac{\partial W_t^B}{\partial U^A}} &\geq \frac{\lambda_t^A}{\lambda_t^B}, \end{aligned}$$

a contradiction. A similar reasoning holds for the second part of the Lemma. ■

Let us now consider the three relevant cases that were also considered in the proof of Proposition 1:

Case (i) In this case, we set $\theta_t^A = \theta_t^B = 1$ and we define:

$$Y_t^A = \mathbf{p}_t' \mathbf{q}_t^A + (\mathbf{P}_t' - \boldsymbol{\tau}_t^{B'}) \mathbf{Q}_t^A + \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B.$$

To obtain a contradiction, let us consider an allocation $(\mathbf{q}^A, \mathbf{Q}^A)$ such that

$$\mathbf{p}_t' \mathbf{q}^A + (\mathbf{P}_t' - \boldsymbol{\tau}_t^{B'}) \mathbf{Q}^A + \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B \leq Y_t^A$$

and,

$$U^A(\mathbf{q}^A, \mathbf{Q}^A + \mathbf{Q}_t^B) > U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B).$$

Denote by $U_{\mathbf{q}_t^A}^A$ and $U_{\mathbf{Q}_t}^A$ the subgradient vectors for U^A with respect to \mathbf{q}^A and \mathbf{Q} at the bundles $(\mathbf{q}_t^A, \mathbf{Q}_t)$. Then, by concavity of U^A , we have that:

$$\begin{aligned} U^A(\mathbf{q}^A, \mathbf{Q}^A + \mathbf{Q}_t^B) - U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B) &\leq U_{\mathbf{q}_t^A}^A(\mathbf{q}^A - \mathbf{q}_t^A) + U_{\mathbf{Q}_t}^A(\mathbf{Q}^A - \mathbf{Q}_t^A) \\ &= \frac{\lambda_t^A}{\frac{\partial W_t^A}{\partial U_t^A}} [\mathbf{p}_t'(\mathbf{q}^A - \mathbf{q}_t^A) + (\mathbf{P}_t' - \boldsymbol{\tau}_t^{B'}) (\mathbf{Q}^A - \mathbf{Q}_t^A)] \\ &\leq 0. \end{aligned}$$

The first inequality follows from Lemma 1 and the fact that condition (5) must hold with equality for case (i). The second inequality follows from the budget constraint and gives us the desired contradiction.

Case (ii) In this case, we define θ_t^B and θ_t^A as in conditions (11) and (12) and we define Y_t^A by

$$Y_t^A = \mathbf{p}_t' \mathbf{q}_t^A + (\mathbf{P}_t' - \theta_t^B \boldsymbol{\tau}_t^{B'}) \mathbf{Q}_t^A + \theta_t^A \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B.$$

One can easily see that for case (ii), $Q_{t,k}^A > 0$ implies that $\tau_k^A(\mathbf{q}_t^A, Q_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) = P_{t,k}$, and by negation, $\tau_k^A(\mathbf{q}_t^A, Q_t) + \theta_t^B \tau_k^B(\mathbf{q}_t^B, \mathbf{Q}_t) < P_{t,k}$ implies $Q_{t,k}^A = 0$. This implies that for all $\mathbf{Q}^A \geq 0$:

$$U_{\mathbf{Q}_t}^A(\mathbf{Q}^A - \mathbf{Q}_t^A) \leq \frac{\lambda_t^A}{\frac{\partial W_t^A}{\partial U_t^A}} (\mathbf{P}_t' - \theta_t^B \boldsymbol{\tau}_t^{B'}) (\mathbf{Q}^A - \mathbf{Q}_t^A)$$

Now, assume on the contrary that there exist an allocation $(\mathbf{q}^A, \mathbf{Q}^A)$ such that

$$\mathbf{p}_t' \mathbf{q}^A + (\mathbf{P}_t' - \theta_t^B \boldsymbol{\tau}_t^{B'}) \mathbf{Q}^A + \theta_t^A \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B \leq Y_t^A$$

and,

$$U^A(\mathbf{q}^A, \mathbf{Q}^A + \mathbf{Q}_t^B) > U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B).$$

Then, by concavity of U^A , we have that:

$$\begin{aligned} U^A(\mathbf{q}^A, \mathbf{Q}^A + \mathbf{Q}_t^B) - U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B) &\leq U_{\mathbf{q}_t^A}^A(\mathbf{q}^A - \mathbf{q}_t^A) + U_{\mathbf{Q}_t}^A(\mathbf{Q}^A - \mathbf{Q}_t^A) \\ &= \frac{\lambda_t^A}{\frac{\partial W_t^A}{\partial U_t^A}} [\mathbf{p}_t'(\mathbf{q}^A - \mathbf{q}_t^A) + (\mathbf{P}_t' - \theta_t^B \boldsymbol{\tau}_t^{B'}) (\mathbf{Q}^A - \mathbf{Q}_t^A)] \\ &\leq 0 \end{aligned}$$

Again, we have a contradiction.

Case (iii) For this last case, we define $\theta_t^A = \theta_t^B = 1$, and,

$$Y_t^A = \mathbf{p}_t' \mathbf{q}_t^A + (\mathbf{P}_t' - \boldsymbol{\tau}_t^{B'}) \mathbf{Q}_t^A + \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B.$$

Assume, on the contrary, that there exist an allocation $(\mathbf{q}^A, \mathbf{Q}^A)$ such that

$$\mathbf{p}_t' \mathbf{q}^A + (\mathbf{P}_t' - \boldsymbol{\tau}_t^{B'}) \mathbf{Q}^A + \boldsymbol{\tau}_t^{A'} \mathbf{Q}_t^B \leq Y_t^A$$

and,

$$U^A(\mathbf{q}^A, \mathbf{Q}^A + \mathbf{Q}_t^B) > U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B).$$

Again, by concavity of U^A , we have that:

$$\begin{aligned} U^A(\mathbf{q}^A, \mathbf{Q}^A + \mathbf{Q}_t^B) - U^A(\mathbf{q}_t^A, \mathbf{Q}_t^A + \mathbf{Q}_t^B) &\leq U_{\mathbf{q}_t^A}^A(\mathbf{q}^A - \mathbf{q}_t^A) + U_{\mathbf{Q}_t^A}^A(\mathbf{Q}^A - \mathbf{Q}_t^A) \\ &= \frac{\lambda_t^B}{\frac{\partial W_t^B}{\partial U_t^A}} [\mathbf{p}_t'(\mathbf{q}^A - \mathbf{q}_t^A) + (\mathbf{P}_t' - \boldsymbol{\tau}_t^{B'}) (\mathbf{Q}^A - \mathbf{Q}_t^A)] \\ &\leq 0 \end{aligned}$$

The equality follows from Lemma (1) and the fact that condition (6) must hold with equality for case (iii).

This concludes the proof for individual A. The proof for individual B is analogous.

Proof of statement 2

Now assume that for each decision situation t there exist indices $\theta_t^A, \theta_t^B \in [0, 1]$ and incomes Y_t^A, Y_t^B such that $\{\mathbf{q}_t^A, \mathbf{q}_t^B, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ satisfies the definition of an equilibrium with transfers for utility functions U^A, U^B . We need to show that there exist caring functions W_t^A and W_t^B satisfying Assumption SC and consumption bundles $\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}$ (with $\mathbf{q}_t^A = \mathbf{q}_t^{A,A} + \mathbf{q}_t^{A,B}$ and $\mathbf{q}_t^B = \mathbf{q}_t^{B,A} + \mathbf{q}_t^{B,B}$) such that $\{\mathbf{q}_t^{A,A}, \mathbf{q}_t^{A,B}, \mathbf{q}_t^{B,B}, \mathbf{q}_t^{B,A}, \mathbf{Q}_t^A, \mathbf{Q}_t^B\}$ is a household equilibrium with caring.

We define the caring functions $W_t^A(U^A, U^B) = U^A + \theta_t^B (\mu_t^A / \mu_t^B) U^B$ and $W_t^B(U^B, U^A) = U^B + \theta_t^A (\mu_t^B / \mu_t^A) U^A$. In this construction, μ_t^A and μ_t^B represent the marginal utilities of the numeraire for members A and B at equilibrium (i.e. $\frac{\partial U^A}{\partial q_1^A} = \mu_t^A$ and $\frac{\partial U^B}{\partial q_1^B} = \mu_t^B$). It is easy to see that these specifications satisfy Assumption SC as long as θ_t^A and θ_t^B are contained in the unit interval. Further, we choose $\mathbf{q}_t^{A,A} = \mathbf{q}_t^A$, $\mathbf{q}_t^{B,B} = \mathbf{q}_t^B$, $\mathbf{q}_t^{A,B} = 0$ and $\mathbf{q}_t^{B,A} = 0$. Let us focus on member A and assume on the

contrary that there exist bundles $\mathbf{q}^{A,A}, \mathbf{q}^{A,B}, \mathbf{Q}^A$ such that

$$\mathbf{p}'_t(\mathbf{q}^{A,A} + \mathbf{q}^{A,B} + \mathbf{q}_t^{B,B}) + \mathbf{P}'_t(\mathbf{Q}^A + \mathbf{Q}_t^B) \leq Y_t,$$

and,

$$U^A(\mathbf{q}^{A,A}, \mathbf{Q}^A + \mathbf{Q}_t^B) + \theta_t^B(\mu_t^A/\mu_t^B)U^B(\mathbf{q}_t^B + \mathbf{q}^{A,B}, \mathbf{Q}^A + \mathbf{Q}_t^B) > \\ U^A(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B(\mu_t^A/\mu_t^B)U^B(\mathbf{q}_t^B, \mathbf{Q}_t).$$

This gives,

$$\begin{aligned} & U^A(\mathbf{q}^{A,A}, \mathbf{Q}^A + \mathbf{Q}_t^B) + \theta_t^B(\mu_t^A/\mu_t^B)U^B(\mathbf{q}_t^B + \mathbf{q}^{A,B}, \mathbf{Q}^A + \mathbf{Q}_t^B) \\ & - U^A(\mathbf{q}_t^A, \mathbf{Q}_t) - \theta_t^B(\mu_t^A/\mu_t^B)U^B(\mathbf{q}_t^B, \mathbf{Q}_t) \\ & \leq U_{\mathbf{q}_t^A}^{A'}(\mathbf{q}^{A,A} - \mathbf{q}_t^A) + U_{\mathbf{Q}_t}^{A'}(\mathbf{Q}^A - \mathbf{Q}_t^A) + \theta_t^B(\mu_t^A/\mu_t^B) \left[U_{\mathbf{q}_t^B}^{B'}\mathbf{q}^{A,B} + U_{\mathbf{Q}_t}^{B'}(\mathbf{Q}^A - \mathbf{Q}_t^A) \right] \\ & = \mu_t^A \left[\mathbf{p}'_t(\mathbf{q}^{A,A} - \mathbf{q}_t^A) + \theta_t^B \mathbf{p}'_t \mathbf{q}^{A,B} + (\boldsymbol{\tau}^{A'}(\mathbf{q}_t^A, \mathbf{Q}_t) + \theta_t^B \boldsymbol{\tau}^{B'}(\mathbf{q}_t^B, \mathbf{Q}_t)) (\mathbf{Q}^A - \mathbf{Q}_t^A) \right] \\ & \leq \mu_t^A \left[\mathbf{p}'_t(\mathbf{q}^{A,A} + \mathbf{q}^{A,B} - \mathbf{q}_t^A) + \mathbf{P}'_t(\mathbf{Q}^A - \mathbf{Q}_t^A) \right] \\ & \leq 0. \end{aligned}$$

The first inequality follows from concavity of the functions U^A and U^B . The first equality follows from the first order conditions of programs **DOP-A** and **DOP-B** for the private goods. The second inequality follows from the fact that $\theta_t^B \leq 1$, the first order conditions of **DOP-A** for the public goods and the fact that $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B < P_{t,k}$ only if $Q_{t,k}^A = 0$.

Proof of Proposition 3

1 \Rightarrow 2. The data set $S = \{\mathbf{p}_t, \mathbf{P}_t, \mathbf{q}_t, \mathbf{Q}_t\}_{t \in T}$ is rationalizable with caring. Because of Proposition 2, we have for any decision situation t that the household allocation solves **DOP-A** and **DOP-B**. As before, let $U_{\mathbf{q}_t^M}^M$ and $U_{\mathbf{Q}_t}^M$ ($M = A, B$) be the subgradients for the function U^M at bundle $(\mathbf{q}_t^M, \mathbf{Q}_t)$, and λ_t^A and λ_t^B the Lagrange multipliers for the budget constraints. We get as first order conditions, for each private good j and public good k ,

$$\begin{aligned} U_{q_{t,j}^A}^A & \leq \lambda_t^A p_{t,j}, \\ U_{q_{t,j}^B}^B & \leq \lambda_t^B p_{t,j}, \\ U_{Q_{t,k}^A}^A & \leq \lambda_t^A (P_{t,k} - \theta_t^B \tau^B(\mathbf{q}_t^B, \mathbf{Q}_t)), \\ U_{Q_{t,k}^B}^B & \leq \lambda_t^B (P_{t,k} - \theta_t^A \tau^A(\mathbf{q}_t^A, \mathbf{Q}_t)). \end{aligned}$$

The inequalities are replaced by equalities in case the quantities of the goods under consideration are strictly positive. Next, concavity of the utility functions U^A and U^B implies, for all decision situations t, v

$$\begin{aligned} U^A(\mathbf{q}_t^A, \mathbf{Q}_t) - U^A(\mathbf{q}_v^A, \mathbf{Q}_v) &\leq U_{\mathbf{q}_v^A}^{A'}(\mathbf{q}_t^A - \mathbf{q}_v^A) + U_{\mathbf{Q}_v}^{A'}(\mathbf{Q}_t - \mathbf{Q}_v), \\ U^B(\mathbf{q}_t^B, \mathbf{Q}_t) - U^B(\mathbf{q}_v^B, \mathbf{Q}_v) &\leq U_{\mathbf{q}_v^B}^{B'}(\mathbf{q}_t^B - \mathbf{q}_v^B) + U_{\mathbf{Q}_v}^{B'}(\mathbf{Q}_t - \mathbf{Q}_v). \end{aligned}$$

For all t , define $U_{\mathbf{q}_t^A}^A/\lambda_t^A = \tau_t^A$ and $U_{\mathbf{q}_t^B}^B/\lambda_t^B = \tau_t^B$, $U^A(\mathbf{q}_t^A, \mathbf{Q}_t) = U_t^A$ and $U^B(\mathbf{q}_t^B, \mathbf{Q}_t) = U_t^B$. This gives,

$$U_t^A - U_v^A \leq \lambda_v^A (\mathbf{p}'_v(\mathbf{q}_t^A - \mathbf{q}_v^A) + \tau_t^A(\mathbf{Q}_t - \mathbf{Q}_v)), \quad (15)$$

$$U_t^B - U_v^B \leq \lambda_v^B (\mathbf{p}'_v(\mathbf{q}_t^B - \mathbf{q}_v^B) + \tau_t^B(\mathbf{Q}_t - \mathbf{Q}_v)). \quad (16)$$

To see that this obtains S.3, we make use of the Afriat Theorem (see Afriat (1967) and Varian (1982)). Specifically, the inequalities in (15)-(16) are so-called Afriat inequalities, and the Afriat Theorem implies that these inequalities are satisfied for all t, v if and only if the sets $\{\mathbf{p}_t, \tau_t^A, \mathbf{q}_t^A, \mathbf{Q}_t\}_{t \in T}$ and $\{\mathbf{p}_t, \tau_t^B, \mathbf{q}_t^B, \mathbf{Q}_t\}_{t \in T}$ satisfy GARP.

Moreover, at the equilibrium, if $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B < P_{t,k}$, then $Q_{t,k}^A = 0$ and, thus, $Q_{t,k}^B = Q_{t,k} > 0$. Then, the first order condition for k in **DOP-B** must be binding, so that $\theta_t^A \tau_{t,k}^A + \tau_{t,k}^B = P_{t,k}$. This obtains the first part of S.2. Reversing the roles of A and B shows the other part of S.2. Similarly, one can verify S.4 and S.5.

2 \Rightarrow 1. Because the GARP conditions in (S.3) are satisfied, the Afriat Theorem (mentioned above) tells us that there exist positive numbers U_t^A, U_t^B and strictly positive numbers λ_t^A and λ_t^B such that the following Afriat inequalities hold:

$$\begin{aligned} U_t^A - U_v^A &\leq \lambda_v^A (\mathbf{p}'_v(\mathbf{q}_t^A - \mathbf{q}_v^A) + \tau_v^{A'}(\mathbf{Q}_t - \mathbf{Q}_v)), \\ U_t^B - U_v^B &\leq \lambda_v^B (\mathbf{p}'_v(\mathbf{q}_t^B - \mathbf{q}_v^B) + \tau_v^{B'}(\mathbf{Q}_t - \mathbf{Q}_v)). \end{aligned}$$

Then, define the functions U^A and U^B such that:

$$\begin{aligned} U^A(\mathbf{q}^A, \mathbf{Q}) &= \min_{v \in T} \{U_v^A + \lambda_v^A (\mathbf{p}'_v(\mathbf{q}^A - \mathbf{q}_v^A) + \tau_v^{A'}(\mathbf{Q} - \mathbf{Q}_v))\}, \\ U^B(\mathbf{q}^B, \mathbf{Q}) &= \min_{v \in T} \{U_v^B + \lambda_v^B (\mathbf{p}'_v(\mathbf{q}^B - \mathbf{q}_v^B) + \tau_v^{B'}(\mathbf{Q} - \mathbf{Q}_v))\}. \end{aligned}$$

Notice that U^A and U^B are continuous, concave, strictly monotone and that for all $t \in T$, $U^A(\mathbf{q}_t^A, \mathbf{Q}_t) = U_t^A$ and $U^B(\mathbf{q}_t^B, \mathbf{Q}_t) = U_t^B$. See, for example, Varian (1982).

We need to show that the functions U^A and U^B provide a rationalization of the data set. For brevity, we only provide the argument for U^A , but a straightforwardly

analogous reasoning applies to U^B . For all $t \in T$, define \mathbf{Q}_t^A and \mathbf{Q}_t^B so that if $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B < P_{t,k}$ then $Q_{t,k}^A = 0$ and $Q_{t,k}^B = Q_{t,k}$, and if $\theta_t^A \tau_{t,k}^A + \tau_{t,k}^B < P_{t,k}$ then $Q_{t,k}^B = 0$ and $Q_{t,k}^A = Q_{t,k}$ (see S.4 and S.5). (If $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B = P_{t,k}$ and $\theta_t^A \tau_{t,k}^A + \tau_{t,k}^B = P_{t,k}$ then we can randomly allocate $Q_{t,k}$ between $Q_{t,k}^A$ and $Q_{t,k}^B$.) Next, consider $t \in T$ and a bundle $(\mathbf{q}^A, \mathbf{Q}^A)$ with $\mathbf{Q} = \mathbf{Q}^A + \mathbf{Q}_t^B$ such that

$$\begin{aligned}
& \mathbf{p}'_t \mathbf{q}^A + \sum_k [(P_{t,k} - \theta_t^B \tau_{t,k}^B) Q_k^A + \theta_t^A \tau_{t,k}^A Q_{t,k}^B] \\
& \leq \mathbf{p}'_t \mathbf{q}_t^A + \sum_k [(P_{t,k} - \theta_t^B \tau_{t,k}^B) Q_{t,k}^A + \theta_t^A \tau_{t,k}^A Q_{t,k}^B] \\
& \text{or} \\
& \mathbf{p}'_t \mathbf{q}^A + \sum_k [(P_{t,k} - \theta_t^B \tau_{t,k}^B) Q_k^A] \\
& \leq \mathbf{p}'_t \mathbf{q}_t^A + \sum_k [(P_{t,k} - \theta_t^B \tau_{t,k}^B) Q_{t,k}^A]. \tag{17}
\end{aligned}$$

Then, we have to prove that $U^A(\mathbf{q}^A, \mathbf{Q}) \leq U^A(\mathbf{q}_t^A, \mathbf{Q}_t)$. To obtain this result, we first note that, by construction, $\tau_t^A \mathbf{Q}_t^A = (\mathbf{P}_t - \theta_t^B \tau_t^B) \mathbf{Q}_t^A$. Thus, because $\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B \leq P_{t,k}$ (which implies $\tau_t^A \mathbf{Q}^A \leq (\mathbf{P}_t - \theta_t^B \tau_t^B) \mathbf{Q}^A$), we get $\tau_t^A (\mathbf{Q}^A - \mathbf{Q}_t^A) \leq (\mathbf{P}_t - \theta_t^B \tau_t^B)' (\mathbf{Q}^A - \mathbf{Q}_t^A)$. Using this, we obtain

$$\begin{aligned}
& U^A(\mathbf{q}^A, \mathbf{Q}) \\
& = \min_{v \in T} \{U_v^A + \lambda_v^A (\mathbf{p}'_v (\mathbf{q}^A - \mathbf{q}_v^A) + \tau_v^{A'} (\mathbf{Q} - \mathbf{Q}_v))\} \\
& \leq U_t^A + \lambda_t^A (\mathbf{p}'_t (\mathbf{q}^A - \mathbf{q}_t^A) + \tau_t^{A'} (\mathbf{Q} - \mathbf{Q}_t)) \\
& = U_t^A + \lambda_t^A (\mathbf{p}'_t (\mathbf{q}^A - \mathbf{q}_t^A) + \tau_t^{A'} (\mathbf{Q}^A - \mathbf{Q}_t^A)) \\
& \leq U_t^A + \lambda_t^A (\mathbf{p}'_t (\mathbf{q}^A - \mathbf{q}_t^A) + (\mathbf{P}_t - \theta_t^B \tau_t^B)' (\mathbf{Q}^A - \mathbf{Q}_t^A)) \\
& \leq U_t^A.
\end{aligned}$$

This provides the wanted result, i.e. $\{\mathbf{q}_t^A, \mathbf{Q}_t^A\}$ solves **DOP-A**.

Appendix B: mixed integer characterization

In this appendix, we reformulate the conditions in Proposition 3 in mixed integer programming (MIP) terms. To obtain this MIP formulation, we define the binary variables $x_{t,v}^M \in \{0, 1\}$, with $x_{t,v}^M = 1$ interpreted as $(\mathbf{q}_t^M, \mathbf{Q}_t) R^M (\mathbf{q}_v^M, \mathbf{Q}_v)$ where R^M is the revealed preference relation for individual $M \in \{A, B\}$. Then, a data set S satisfies the necessary and sufficient condition for rationalizability as given by Proposition 3 if and only if the following MIP problem is feasible:

For all decision situations t, v and public goods k there exist strictly positive vectors $\tau_t^A, \tau_t^B \in \mathbb{R}_{++}^K$, binary variables $z_{t,k}, x_{t,v}^A, x_{t,v}^B \in \{0, 1\}$, and parameters $\theta_t^A, \theta_t^B \in [0, 1]$

such that:²⁴

$$\tau_t^A + \theta_t^B \tau_t^B \leq \mathbf{P}_t, \quad (\text{M.1})$$

$$\theta_t^A \tau_t^A + \tau_t^B \leq \mathbf{P}_t, \quad (\text{M.2})$$

$$P_{t,k} - \tau_{t,k}^A - \theta_t^B \tau_{t,k}^B \leq z_{t,k} C_t, \quad (\text{M.3})$$

$$P_{t,k} - \theta_t^A \tau_{t,k}^A - \tau_{t,k}^B \leq (1 - z_{t,k}) C_t, \quad (\text{M.4})$$

$$\mathbf{q}_t^A + \mathbf{q}_t^B = \mathbf{q}_t, \quad (\text{M.5})$$

$$\mathbf{p}'_t(\mathbf{q}_t^M - \mathbf{q}_v^M) + \tau_t^{M'}(\mathbf{Q}_t - \mathbf{Q}_v) \leq x_{t,v}^M C_t, \quad (\text{M.6})$$

$$x_{t,s}^M + x_{s,v}^M \leq 1 + x_{t,v}^M, \quad (\text{M.7})$$

$$(1 - x_{t,v}^M) C_v \geq \mathbf{p}'_v(\mathbf{q}_v^M - \mathbf{q}_t^M) + \tau_v^{M'}(\mathbf{Q}_v - \mathbf{Q}_t), \quad (\text{M.8})$$

(18)

with C_t a given number for which $C_t > P_{t,k}$ and $C_t > Y_t$ for all t, k .

The explanation is as follows. Constraint (M.5) imposes that the private consumption bundles \mathbf{q}_t^A and \mathbf{q}_t^B sum to the observed aggregate quantities \mathbf{q}_t , as required by condition S.1. Further, constraints M.1-M.4 comply with condition S.2 in Proposition 3. Specifically, M.1 and M.2 impose the given upper bound restriction for τ_t^A and τ_t^B . Next, M.3 imposes $P_{t,k} \leq \tau_{t,k}^A + \theta_t^B \tau_{t,k}^B$ if $z_{t,k} = 0$, while M.4 imposes $P_{t,k} \leq \theta_t^A \tau_{t,k}^A + \tau_{t,k}^B$ if $z_{t,k} = 1$. Because $z_{t,k} \in \{0, 1\}$, this implies $\max\{\tau_{t,k}^A + \theta_t^B \tau_{t,k}^B, \tau_{t,k}^B + \theta_t^A \tau_{t,k}^A\} = P_{t,k}$ and thus condition S.2 is satisfied. Finally, constraints M.6-M.8 correspond to the GARP conditions for each individual M ($= A$ or B) (condition S.3 in Proposition 3). Specifically, M.6 states that $\mathbf{p}'_t(\mathbf{q}_t^M - \mathbf{q}_v^M) + \tau_t^{M'}(\mathbf{Q}_t - \mathbf{Q}_v) \geq 0$ implies $x_{t,v}^M = 1$ (or $(\mathbf{q}_t^M, \mathbf{Q}_t) R^M (\mathbf{q}_v^M, \mathbf{Q}_v)$). Next, constraint M.7 imposes transitivity of the individual revealed preference relations R^M : if $x_{t,s}^M = 1$ (i.e. $(\mathbf{q}_t^M, \mathbf{Q}_t) R^M (\mathbf{q}_s^M, \mathbf{Q}_s)$) and $x_{s,v}^M = 1$ (i.e. $(\mathbf{q}_s^M, \mathbf{Q}_s) R^M (\mathbf{q}_v^M, \mathbf{Q}_v)$) then $x_{t,v}^M = 1$ (i.e. $(\mathbf{q}_t^M, \mathbf{Q}_t) R^M (\mathbf{q}_v^M, \mathbf{Q}_v)$). And M.8 requires $\mathbf{p}'_v(\mathbf{q}_v^M - \mathbf{q}_t^M) + \tau_v^{M'}(\mathbf{Q}_v - \mathbf{Q}_t) \leq 0$ if $x_{t,v}^M = 1$ (i.e. $(\mathbf{q}_t^M, \mathbf{Q}_t) R^M (\mathbf{q}_v^M, \mathbf{Q}_v)$).

Clearly, all constraints are linear for fixed θ_t^A and θ_t^B . Linearity implies that the above program can be solved by standard MIP methods for a given data set S . See also our discussion in the main text on conducting a grid search for $\theta_t^A, \theta_t^B \in [0, 1]$.

References

- [1] Afriat, S. (1967), The Construction of Utility Functions from Expenditure Data, *International Economic Review* 8, 67–77.

²⁴The strict inequality $\mathbf{p}'_t(\mathbf{q}_t^M - \mathbf{q}_v^M) + \tau_t^{M'}(\mathbf{Q}_t - \mathbf{Q}_v) < x_{t,v}^M C_t$ is difficult to use in IP analysis. Therefore, in practice we can replace it with $\mathbf{p}'_t(\mathbf{q}_t^M - \mathbf{q}_v^M) + \tau_t^{M'}(\mathbf{Q}_t - \mathbf{Q}_v) + \epsilon \leq x_{t,v}^M C_t$ for $\epsilon (> 0)$ arbitrarily small.

- [2] Alderman, H., P.-A. Chiappori, L. Haddad, J. Hoddinott and R. Kanbur (1995), Unitary versus collective models of the household: is it time to shift the burden of proof?, *World Bank Research Observer* 10, 1-19.
- [3] Andreoni, J. and W.T. Harbaugh (2006), Power indices for revealed preference tests, University of Wisconsin-Madison Department of Economics Working Paper, 2005-10, version March 2006.
- [4] Apps, P. and Rees, R. (1988), Taxation and the Household, *Journal of Public Economics* 35, 355-369.
- [5] Apps, P. and Rees, R. (2009), Public economics and the household, *Cambridge University Press*.
- [6] Beatty, T. and Crawford, I. (2010), How Demanding is the Revealed Preference approach to Demand?, *American Economic Review*, forthcoming.
- [7] Becker, G. (1981), Altruism in the family and selfishness in the market, *Economica* 48, 1-15.
- [8] Bergstrom, T. (1976), Theory and measurement of economic externalities, *Academic Press*.
- [9] Bergstrom, T., Blume, L. and Varian, H. (1986), On the private provision of public goods, *Journal of Public Economics* 29, 25-49.
- [10] Bronars, S. (1987), The power of nonparametric tests of preference maximization, *Econometrica* 55, 693-698.
- [11] Blundell, R., M. Browning and I. Crawford (2003), Nonparametric Engel Curves and. Revealed Preference, *Econometrica* 71, 205-240.
- [12] Blundell, R., M. Browning and I. Crawford (2008), Best nonparametric bounds on demand responses, *Econometrica* 76, 1227-1262.
- [13] Blundell, R., P.-A. Chiappori and C. Meghir (2005), Collective labor supply with children, *Journal of Political Economy* 113, 1277-1306.
- [14] Bourguignon, F. (1984), Rationalité individuelle our rationalité stratégique: le cas de l'offre familiale de travail, *Revue Economique* 35, 147-162.
- [15] Bourguignon, F., M. Browning, P.-A. Chiappori (2009), Efficient intrahousehold allocations and distribution factors: implications and identification, *Review of Economic Studies* 76, 503-528.

- [16] Bourguignon, F., M. Browning, P.-A. Chiappori and V. Lechene (1993), Intra-household allocation of consumption: a model and some evidence from French data, *Annales d'Economie et de Statistique* 29, 137-156.
- [17] Browning, M. (1989), A nonparametric test of the life-cycle rational expectations hypothesis, *International Economic Review* 30, 979-992.
- [18] Browning, M. (2000), The saving behaviour of a two-person household, *Scandinavian Journal of Economics* 102, 235-251
- [19] Browning, M, F. Bourguignon, P.-A. Chiappori and V. Lechene (1994), A structural model of intrahousehold allocation, *Journal of Political Economy* 102, 1067-1096.
- [20] Browning, M. and Chiappori, P.-A. (1998), Efficient Intra-houshold Allocations: A General Characterisation and Empirical Tests, *Econometrica* 66, 1241–1278.
- [21] Browning, M, P.-A. Chiappori and V. Lechene (2006), Collective and Unitary Models: A Clarification, *Review of Economics of the Household*, 4, 5-14.
- [22] Browning, M, P.-A. Chiappori and V. Lechene (2010), Distributional effects in household models: separate spheres and income pooling, *Economic Journal* 120, 786-799.
- [23] Browning, M., P.-A. Chiappori and A. Lewbel (2006), Estimating consumption economies of scale, adult equivalence scales, and household bargaining power, Boston College Working Paper WP588, Department of Economics, Boston College, version 2006.
- [24] Browning, M. and V. Lechene (2001), Caring and sharing: tests between alternative models of intra-household allocation, University of Oxford discussion paper 70, Department of Economics.
- [25] Chen, Z. and F. Woolley (2001), A Cournot-Nash equilibrium of family decision making, *Economic Journal* 111, 722-748.
- [26] Cherchye, L., T. Demuynck and B. De Rock (2011), Revealed preference analysis of noncooperative household consumption, *Economic Journal*, forthcoming.
- [27] Cherchye, L., B. De Rock and F. Vermeulen (2007), The collective model of household consumption: a nonparametric characterization, *Econometrica* 75, 553-574.
- [28] Cherchye, L., B. De Rock, and F. Vermeulen (2009), Opening the black box of intra-household decision-making: theory and non-parametric empirical tests

- of general collective consumption models, *Journal of Political Economy* 117, 1074-1104.
- [29] Cherchye, L., B. De Rock and F. Vermeulen (2011), The revealed preference approach to collective consumption behavior: testing and sharing rule recovery, *Review of Economic Studies* 78, 176–198..
 - [30] Cherchye, L. and F. Vermeulen (2008), Nonparametric analysis of household labor supply: goodness of fit and power of the unitary and the collective model, *Review of Economics and Statistics* 90, 267-274.
 - [31] Chiappori, P.-A. (1988), Rational household labor supply, *Econometrica* 56, 63-89.
 - [32] Chiappori, P.-A. (1992), Collective labor supply and welfare, *Journal of Political Economy* 100, 437-467.
 - [33] Chiappori, P.-A. and I. Ekeland (2006), The micro economics of group behavior: general characterization, *Journal of Economic Theory* 130, 1–26.
 - [34] Chiappori, P.-A. and I. Ekeland (2009), The micro economics of efficient group behavior: identification, *Econometrica* 77, 763-799.
 - [35] Chiappori, P.-A., B. Fortin and G. Lacroix (2002), Marriage market, divorce legislation and, and household labor supply, *Journal of Political Economy* 110, 37-72.
 - [36] Crawford, I. (2010), Habits Revealed, *Review of Economic Studies* 77, 1382-1402.
 - [37] d' Aspremont, C. and Dos Santos Ferreira, R. (2009), Household Behavior and Individual Autonomy, CORE working paper, Louvain-la-Neuve.
 - [38] Donni O. (2008), Collective models of the household, *The New Palgrave, Dictionary of Economics, second edition*, London: Palgrave McMillan.
 - [39] Donni, O. (2009), A Simple Approach to Investigate Intrahousehold Allocation of Private and Public Goods, *Review of Economics and Statistics* 91, 617-628.
 - [40] Diewert, W. E. (1973), Afriat and revealed preference theory, *Review of Economic Studies* 40, 419–426.
 - [41] Duflo, E. (2003), Grandmothers and granddaughters: old age pension and intra-household allocation in South Africa, *World Bank Economic Review* 17, 1-25.

- [42] Fortin, B. and G. Lacroix (1997), A test of the unitary and collective models of household labour supply, *Economic Journal* 107, 933-955.
- [43] Houthakker, H. S. (1950), Revealed preference and the utility function, *Economica* 17, 159-174.
- [44] Kemp, M.C. (1984), A note of the theory of international transfers, *Economics Letters* 14, 259-262.
- [45] Kooreman, P. and A. Kapteyn (1990), On the empirical implementation of some game theoretic models of household labor supply, *Journal of Human Resources* 25, 584-598.
- [46] Lechene, V., Preston, I. (2005), Household Nash Equilibrium with Voluntary Contributed Public Goods, IFS working paper 05/06.
- [47] Lechene, V., Preston, I. (2008), Non Cooperative Household Demand, *Journal of Economic Theory*, forthcoming.
- [48] Leuthold, J (1968), An empirical study of formula transfers and the work decision of the poor, *Journal of Human Resources* 3, 312-323.
- [49] Lewbel, A. and K. Pendakur (2008), Estimation of Collective Household Models with Engel Curves, *Journal of Econometrics* 147, 350-358.
- [50] Lundberg, S. (1988), Labor supply of husbands and wives: a simultaneous equations approach, *Review of Economics and Statistics* 70, 224-235.
- [51] Lundberg, S. and R. Pollak (1993), Separate Spheres Bargaining and the Marriage Market, *Journal of Political Economy* 101, 988-1010.
- [52] Lundberg, S., R. Pollak and T. Wales (1997), Do husbands and wives pool their resources? Evidence from the UK child benefit, *Journal of Human Resources* 32, 463-480.
- [53] Manser, M. and M. Brown (1980), Marriage and household decision making: a bargaining analysis, *International Economic Review* 21, 31-44.
- [54] Mazzocco, M. (2007), Household Intertemporal Behaviour: A collective characterization and a test of commitment, *Review of Economic Studies* 74, 857-895.
- [55] McElroy, M. B. and M. Horney (1981), Nash-bargained household decisions: towards a generalization of the theory of demand, *International Economic Review* 22, 333-349.

- [56] Phipps, S. and P. Burton (1998), What's mine is yours? The influence of male and female income on patterns of household consumption, *Economica* 65, 599-613.
- [57] Samuelson, P. A. (1938), A Note on the pure theory of consumer behavior, *Economica* 5, 61-71.
- [58] Samuelson, P. A. (1954), The pure theory of public expenditures, *Review of Economics and Statistics* 36, 387-389.
- [59] Thomas, D. (1990), Intra-household resource allocation: an inferential approach, *Journal of Human Resources* 25, 635-664.
- [60] Ulph, D. (1988), A general non-cooperative Nash model of household consumption behavior, University of Bristol, Economics Decision Paper 88/205.
- [61] Varian, H. (1982), The Nonparametric Approach to Demand Analysis, *Econometrica* 50, 945-973.
- [62] Vermeulen, F. (2002), Collective household models: principles and main results, *Journal of Economic Surveys* 16, 533-564.
- [63] Vermeulen, F. (2005), And the winner is... an empirical evaluation of unitary and collective labour supply models, *Empirical Economics* 30, 711-734.

Table 1: test results (1 = pass; 0 = fail) and power for different degrees of intra-household cooperation

Household 1	θ^A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
θ^B	0	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.160	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173
	0.1	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.158	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173
	0.2	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.157	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173
	0.3	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.157	0.157	0.154	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173
	0.4	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.157	0.157	0.155	0.159	0.161	0.168	0.168	0.169	0.169	0.170	0.173
	0.5	Pass (0/1)	1	1	1	1	1	1	1	0	0	0	0
		Power	0.154	0.154	0.154	0.158	0.161	0.168	0.168	0.169	0.169	0.170	0.173
	0.6	Pass (0/1)	1	1	1	1	1	1	0	0	0	0	0
		Power	0.160	0.160	0.160	0.159	0.162	0.169	0.169	0.169	0.169	0.170	0.173
	0.7	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0
		Power	0.170	0.170	0.170	0.170	0.171	0.170	0.169	0.169	0.169	0.170	0.173
	0.8	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0
		Power	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.168	0.169	0.170	0.173
	0.9	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0
		Power	0.170	0.170	0.170	0.170	0.170	0.170	0.170	0.170	0.169	0.170	0.173
	1	Pass (0/1)	0	0	0	0	0	0	0	0	0	0	0
		Power	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173	0.173
Household 2	θ^A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	
θ^B	0	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1
		Power	0.108	0.102	0.102	0.102	0.100	0.104	0.106	0.103	0.105	0.107	0.108
	0.1	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1
		Power	0.108	0.103	0.102	0.102	0.099	0.103	0.105	0.103	0.105	0.107	0.108
	0.2	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1
		Power	0.109	0.103	0.101	0.101	0.099	0.103	0.105	0.103	0.105	0.107	0.108
	0.3	Pass (0/1)	0	0	0	0	0	0	1	1	1	1	1
		Power	0.107	0.104	0.101	0.101	0.099	0.102	0.104	0.103	0.105	0.107	0.108
	0.4	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.108	0.107	0.105	0.103	0.101	0.102	0.104	0.103	0.105	0.107	0.108
	0.5	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.107	0.107	0.105	0.103	0.101	0.103	0.105	0.104	0.105	0.107	0.108
	0.6	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.103	0.103	0.104	0.103	0.101	0.103	0.104	0.104	0.105	0.107	0.108
	0.7	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.102	0.102	0.102	0.102	0.102	0.102	0.104	0.104	0.105	0.107	0.108
	0.8	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.105	0.106	0.108
	0.9	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.106	0.105	0.106	0.108
	1	Pass (0/1)	1	1	1	1	1	1	1	1	1	1	1
		Power	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108	0.108